The University of Oxford

Final Honour School of Mathematical and Theoretical Physics Part C
Master of Science Mathematical and Theoretical Physics

String Theory I

Hilary Term 2019

Your task in this project is to develop the basic theory of **bosonic string orbifolds**. The essentials of the orbifold construction can be found in many of the standard references, such as Section 8.5 of Polchinski's *String Theory Volume 1.*

The main emphasis in your report should be the orbifold of a single spacetime dimension, so target spacetimes of the form $\mathbb{R}^{1,24} \times \mathbb{R}/\mathbb{Z}_2$ and $\mathbb{R}^{1,24} \times S^1/\mathbb{Z}_2$. That is, you should consider the (free) bosonic string sigma model subject to the identifications

$$\begin{aligned} \mathbb{R}^1/\mathbb{Z}_2 : &\quad X^{25} \cong -X^{25} \ , \\ S^1/\mathbb{Z}_2 : &\quad X^{25} \cong \pm X^{25} + 2\pi R m \ , \quad m \in \mathbb{Z} \ . \end{aligned}$$

In your report, you should explain in some detail the following:

(i) *Canonical quantization of the closed string mode in an orbifold background.*

(ii) *The notion of twisted sectors and the quantization of twisted sector strings.*

(iii) *Physical state conditions*[1] *and the spectrum of physical states at low levels (at least, say, levels zero and one) in both untwisted and twisted sectors.*

In addition to the basics, you should choose a more sophisticated aspect of the topic to elaborate upon. Two good examples would be:

- Torus amplitudes for orbifold spacetimes and modular invariance. *[For this you should include some general discussion of torus amplitudes and the interpretation of modular invariance. You might include a path integral argument for the inclusion of twisted sectors].*
- The equivalence of the $S^1/\mathbb{Z}_2$ orbifold at $R = \sqrt{\alpha'}$ with the ordinary S^1 compactification at $R = 2\sqrt{\alpha'}$. *[You could detail the enhanced symmetry of bosonic strings at the self-dual radius $R = \sqrt{\alpha'}$ and derive the aforementioned equivalence in these terms].*

If you prefer, you may pursue an alternate topic as long as it is related to the subject matter of bosonic string orbifolds.

Your report should *not* be a simple repetition of the material as presented in standard textbooks. You should carry out some of the computations and include them in greater detail than that presented in the texts. You should try to draw from multiple sources to synthesize a coherent exposition of the subject.

Your report should be ten to fifteen pages in length, in an 11 point font, with one inch margins. You should indicate clearly what ideas come from existing sources and, if relevant, which are original. You should make attributions to all of your sources.

Your report need not contain original research results

[1] You do not need to derive the normal ordering constant in the twisted sector, and can instead just quote the result $a = 15/16$ for the case of a single twisted direction.