Honour School of Mathematical and Theoretical Physics Part C
Master of Science in Mathematical and Theoretical Physics

RADIATIVE PROCESSES AND HIGH-ENERGY ASTROPHYSICS TAKE HOME EXAM

Trinity Term 2021

MONDAY, 14TH JUNE 2021 09:30 am BST to WEDNESDAY, 16TH JUNE 2021 09:30 am BST

You should submit answers to all questions.

You may either type or handwrite your answers. You may refer to books and other sources when completing the exam but should not discuss the exam with anyone else.

The numbers in the margin indicate the weight that the Examiners anticipate assigning to each part of the question.

1. Explain how hydrogen recombination lines form. What transition dominates the bound-free cross-section of hydrogen, and why? Why do recombinations directly to the ground state of hydrogen not contribute any net recombinations towards the balance of ionization and recombination? [2]

   The beachball nebula has a star at its centre with radius r_* and specific luminosity L_ν^* that clears out a cavity of radius $r_{\rm in}$ at the centre of the nebula with a stellar wind. Hydrogen recombination lines are seen from a resolved region of the nebula with angular diameter $\theta = 56$ milliarcseconds. Modelling this region as a Stromgren Sphere, show that its radius can be approximated as

$$r_s = \left(\frac{3\dot{N}_\gamma}{4\pi\alpha_r n_H^2} \right)^{1/3},$$

   where $\dot{N}_\gamma$ is the number of photons per second emitted by the central star, n_H is the hydrogen number density and $\alpha_r = 2.6 \times 10^{-19}$ m^{3}s^{-1} is the recombination coefficient of hydrogen. List your assumptions. [3]

   Forbidden oxygen lines from this region indicate that the electron number density is $n_e = 10^{10}$ m^{-3}, and the flux in the H$_\alpha$ line is observed to be $F_{H_\alpha} = 4.6 \times 10^{-13}$ erg/s/m^2. Calculate the distance, D, in parsecs from the Earth to the beachball nebula. Why may Brackett series line fluxes provide a better estimate of the distance? [5]

   The Stromgren Sphere is a simplified model. In reality, the ionized fraction, $\zeta \equiv n_{HII}/n_H$, is a continuous function of radius. Show that the ionized fraction at the inner edge of the nebula is

$$\zeta_{in} = \frac{-1 + \sqrt{1 + 4\Theta}}{2\Theta},$$

   where

$$\Theta \equiv \frac{n_H \ \alpha_r \ r_*^2 \ hc}{8.8 \ \langle\sigma\rangle \ F_{H_\alpha} \ \lambda_{H_\alpha} \ D^2} \quad \text{and} \quad \langle\sigma\rangle \equiv \frac{1}{\dot{N}_\gamma} \int_0^\infty \sigma_\nu \ \frac{L_\nu^*}{h\nu} \ d\nu.$$

   Here, σ_ν is the hydrogen photo-ionization cross-section. [7]

   Show that the ionized fraction at radius $r = r_{\rm in} + \Delta r$, where $\Delta r \ll r_{\rm in}$, is

$$\zeta \approx \frac{-\Gamma + \sqrt{\Gamma^2 + 4\Theta\Gamma}}{2\Theta},$$

   where

$$\Gamma \equiv 1 - n_H \ \Delta r \ \frac{\langle\sigma^2\rangle}{\langle\sigma\rangle} \ (1 - \zeta_{in}).$$

   [8]

   1 erg $= 10^{-7}$ J; $\lambda_{H_\alpha} = 656.28$ nm; 1 pc $= 3.086 \times 10^{16}$ m; $h = 6.63 \times 10^{-34}$ m^2 kg s^{-1}; $c = 3 \times 10^8$ m s^{-1}.

2. Explain why jet ejecta can have an apparent velocity across the sky greater than the speed of light. Show that a blob of plasma travelling towards the observer with speed $v = \beta c$ and inclination angle θ has an apparent transverse velocity $v_{\rm app} = \beta_{\rm app} c$, where

$$\beta_{\rm app} = \frac{\beta \sin\theta}{1 - \beta\cos\theta}.$$

[5]

What is the minimum value of β required for us to observe apparent superluminal motion? For what inclination angle does this occur? [5]

In 2018, jet ejecta were seen to move away from the black hole X-ray binary system MAXI J1820+070 in radio observations. The approaching jet component moved across the sky at a rate of $\mu_{\rm app} = 77$ milliarcseconds per day, and the receding component at $\mu_{\rm rec} = 33$ milliarcseconds per day. Assuming the same intrinsic speed for the two components, calculate the maximum distance to the source from Earth allowed by causality, $D_{\rm max}$, and the corresponding maximum inclination angle, $\theta_{\rm max}$. [5]

Both jet components have a power-law spectrum, $I_\nu \propto \nu^{-\alpha}$. The approaching component is 24 times the brightness of the receding component in the $\nu = 15.5$ GHz band. Assuming the two components have the same intrinsic intensity, calculate α. Assuming optically thin synchrotron radiation, how does your value of α compare to the expectations of the diffusive shock acceleration model and to the observed cosmic ray spectrum? [5]

The motion of the ejecta on the sky can equally well be fit with an alternative model, whereby the approaching and receding components have initial velocities $\mu_{\rm app,0} = 101$ and $\mu_{\rm rec,0} = 59$ milliarcseconds per day upon launch, and have since been slowing down with constant decelerations 0.5 and 0.33 milliarcseconds per day per day. What are $\theta_{\rm max}$ and $D_{\rm max}$ in this case? The distance to the source has been measured using radio parallax to be $D = 2.96 \pm 0.33$ kpc. Does this favour the constant velocity or constant acceleration model? [5]