Honour School of Mathematical and Theoretical Physics Part C
Master of Science in Mathematical and Theoretical Physics

SUPERSYMMETRY AND SUPERGRAVITY

Trinity Term 2017

FRIDAY, 21ST APRIL 2017, 2.30pm to 5.30pm

You should submit answers to three of the four questions.

You must start a new booklet for each question which you attempt. Indicate on the front sheet the numbers of the questions attempted. A booklet with the front sheet completed must be handed in even if no question has been attempted.

The numbers in the margin indicate the weight that the Examiners anticipate assigning to each part of the question.

Do not turn this page until you are told that you may do so

1. (a) [4 marks] Explain how to construct terms in a Lagrangian invariant under supersymmetry. Describe explicitly, using superspace, how invariant terms in the Lagrangian can be found in $\mathcal{N}=1$ supersymmetry.

   (b) [4 marks] Write down the most general $\mathcal{N}=1$ supersymmetric action in four dimensions involving chiral and Abelian vector superfields. Explain the terms and quantities you have used.

   (c) [3 marks] Comment on how the individual terms in this action behave upon renormalisation.

   (d) [14 marks] Calculate the F-term associated with the term $W^\alpha W_\alpha$, where W_α denotes the field strength superfield for a U(1) gauge group. In components the superfield W_α is given by:

$$W_\alpha = \lambda_\alpha \ + \ \theta_\alpha D \ + \ (\sigma^{\mu\nu})_\alpha{}^\beta \, \epsilon_{\beta\gamma} \, \theta^\gamma \, F_{\mu\nu} \ - \ i\,(\theta\theta)\,(\sigma^\rho)_{\alpha\dot\beta}\,\epsilon^{\dot\beta\dot\delta}\,\partial_\rho \bar\lambda_{\dot\delta} \ .$$

   To simplify your expression you may use the following identity:

$$\mathrm{Tr}(\sigma^\mu\bar\sigma^\nu\sigma^\lambda\bar\sigma^\rho) = 2i\epsilon^{\mu\nu\lambda\rho} + 2\eta^{\mu\nu}\eta^{\lambda\rho} - 2\eta^{\mu\lambda}\eta^{\nu\rho} + 2\eta^{\mu\rho}\eta^{\nu\lambda} \ .$$

2. (a) [6 marks] Determine A in the following equation

$$(\theta\sigma^\mu\bar\theta)(\theta\sigma^\nu\bar\theta) = \frac{A}{2}\eta^{\mu\nu}(\theta\theta)(\bar\theta\bar\theta)$$

   (b) [10 marks] Explain how a general superfield transforms under a SUSY transformation. Outline how the transformation behaviour of different components is obtained. Using the differential operators (justify why you can use them)

$$\begin{aligned} \mathcal{Q}_\alpha &= -i\frac{\partial}{\partial\theta^\alpha} - (\sigma^\mu)_{\alpha\dot\beta}\bar\theta^{\dot\beta}\frac{\partial}{\partial x^\mu} \\ \bar{\mathcal{Q}}_{\dot\alpha} &= i\frac{\partial}{\partial\bar\theta^{\dot\alpha}} - \theta^\beta(\sigma^\mu)_{\beta\dot\alpha}\frac{\partial}{\partial x^\mu} \\ \mathcal{P}^\mu &= -i\partial_\mu \end{aligned}$$

   show explicitly the following changes in the respective parts of the superfield:

$$\begin{aligned} \delta\varphi &= \epsilon\psi + \bar\epsilon\bar\chi \\ \delta\psi &= 2\epsilon M + \sigma^\mu\bar\epsilon(i\partial_\mu\varphi + V_\mu) \end{aligned}$$

   (c) [6 marks] Show that for any superfield S, the superfield $\Phi = \bar{\mathcal{D}}\bar{\mathcal{D}}S$ is chiral. What is the scalar (i.e. $\mathcal{O}(\theta^0,\bar\theta^0)$) component of Φ in terms of the components of S?

   (d) [3 marks] How can chiral superfields be phenomenologically relevant? Comment on the physical role of the component fields. Name at least one advantage and one disadvantage of theories with chiral superfields compared with their non-supersymmetric counterpart.

3. (a) [6 marks] Write down the matter content, with the associated gauge charges, of the minimal supersymmetric Standard Model (MSSM).

   (b) [3 marks] List three different mechanisms of soft supersymmetry breaking for the MSSM and comment on how gaugino masses are generated with these mechanisms.

   (c) [9 marks] Consider the O'Raifertaigh model:

$$\begin{aligned} K &= \Phi_i^\dagger \Phi_i \\ W &= g\Phi_1(\Phi_3^2 - m^2) + M\Phi_2\Phi_3 \end{aligned}$$

   where $M \gg m > 0$ and $g \neq 0$. Show that supersymmetry is necessarily broken in this model. In the limit $m^2 < M^2/(2g^2)$ show that the minimum of the potential is at $V = g^2 m^4$.

   (d) [7 marks] Making the additional assumption that the gauge kinetic functions of the MSSM gauge groups is universal at the high-energy cut-off scale and is given by $f = \varphi_1$ and that the Kähler potential contains a term of the form

$$K = \frac{1}{\Lambda^2}\Phi_1\Phi_1^\dagger \Phi_{\text{MSSM}}\Phi_{\text{MSSM}}^\dagger,$$

   calculate the gaugino and scalar masses from gravity/moduli mediation in this setup. Work in units with $M_P = 1$.

4. (a) [4 marks] Write down the supersymmetry algebra in four dimensions and define the quantities you have used.

   (b) [12 marks] Describe how to construct massive multiplets in a $\mathcal{N} > 1$ theory with vanishing central charges and construct the explicit multiplet based on a ground state with spin $j = 0$ in a theory with $\mathcal{N} = 2$ supersymmetry.

   (c) [5 marks] Write down the $\mathcal{N} = 4$ massless vector-multiplet based on a ground state of helicity $\lambda = -1$ and express it in terms of $\mathcal{N} = 2$ and $\mathcal{N} = 1$ multiplets.

   (d) [4 marks] Show that the total number of states in a given massless multiplet is given by $2^{\mathcal{N}}$ where $\mathcal{N}$ denotes the number of supersymmetries.