A15809S1

Honour School of Mathematical and Theoretical Physics Part C
Master of Science in Mathematical and Theoretical Physics

RADIATIVE PROCESSES AND HIGH ENERGY ASTROPHYSICS

Trinity Term 2017

MONDAY, 12th JUNE 2017, 2:30pm to WEDNESDAY 14th JUNE 2017, 2:30pm

You should submit answers to all questions.

Answer booklets are provided for you to use but you may type your answers if you wish. Typed answers should be printed single-sided and the pages securely fastened together.

You may refer to books and other sources when completing the exam but should not discuss the exam with anyone else.

The numbers in the margin indicate the weight that the Examiners anticipate assigning to each part of the question.

Do not turn this page until you are told that you may do so

1. (a) [4 marks] Describe *two* mechanisms by which very-high-energy gamma-rays (i.e., having energy $E > 10^{12}\,\mathrm{eV}$) may be produced in astrophysical contexts.

   (b) [8 marks] Give a qualitative description of the mechanism by which Cherenkov radiation is produced when a charged particle passes through a dielectric medium at a speed faster than the phase velocity of light in the medium. Show that in a medium of refractive index n, a charged particle travelling with speed $v = \beta c$ will create Cherenkov radiation at an angle θ to the direction of the particle's motion, such that

$$\cos\theta = \frac{1}{n\beta}.$$

   You may assume the medium is non-dispersive.

   (c) [8 marks] A gamma-ray of energy $10^{15}\,\mathrm{eV}$ enters the Earth's atmosphere vertically from space. At an altitude of 20 km above the Earth's surface, it interacts with the atmosphere to produce an electron-positron (e^-/e^+) pair. Further interactions result in a cascade of Cherenkov-producing particles continuing down to an altitude of 10 km, at which point the e^-/e^+ pairs have insufficient energy to produce further Cherenkov light. Taking the refractive index of the atmosphere to have a constant value of $n = 1.0003$, calculate the duration of the Cherenkov light flash from the point of view of an observer on the Earth's surface directly underneath the particle cascade.

   (d) [5 marks] Estimate the area illuminated by the Cherenkov light on the Earth's surface and calculate the average flux of the light during the event within the illuminated region. How does this compare with the flux of light from a nearby star? (Consider a star with a luminosity equal to that of the Sun and at a distance of one parsec from Earth.)

2. (a) [5 marks] Outline the evidence that synchrotron radiation is the source of the radio-frequency emission from active galaxies.

(b) [5 marks] Consider a relativistic electron with Lorentz factor γ orbiting in a magnetic field B, with the plane of the orbit being perpendicular to the field lines. Show that the orbital frequency ν is given by

$$\nu = \frac{eB}{2\pi\gamma m_{\mathrm{e}}} \,,$$

and show that the synchrotron radiation produced by such an electron has a spectrum which is sharply peaked at a characteristic frequency ν_{c} such that

$$\nu_{\mathrm{c}} \sim \frac{\gamma^2 eB}{2\pi m_{\mathrm{e}}} \,.$$

(c) [5 marks] By considering the Lorentz invariance of the power emitted by the electron, or otherwise, show that the total power emitted in synchrotron radiation may be given by

$$P = \frac{2\gamma^2\beta^2 c\sigma_{\mathrm{T}} B^2}{3\mu_0} \,,$$

where β is the speed of the electron relative to the speed of light and σ_{T} is the Thomson cross section. You may assume without proof the Larmor formula for the non-relativistic emitted power from an electron experiencing an acceleration a,

$$P_{\mathrm{Larmor}} = \frac{e^2 a^2}{6\epsilon_0 c^3}$$

Hence calculate a characteristic timescale for the radiative lifetime of the electron.

(d) [10 marks] An active galaxy is observed to have radio lobes with a total length of 4 Mpc, extending symmetrically on either side of the nucleus. Near the nucleus, the radio spectrum exhibits a cutoff in emission at frequencies above 1.0 GHz. Measurements of the total synchrotron brightness of the radio lobes suggest the magnetic field strength is 0.3 nT. Estimate the age of the radio source and the speed at which the radio lobes are advancing into the intergalactic medium, describing the assumptions made and any limitations to the technique.
[Thomson cross section: $\sigma_T = 6.65 \times 10^{-29}$ m^2]