Examiners' Report: Final Honour School of Mathematical and Theoretical Physics Part C and MSc in Mathematical and Theoretical Physics Trinity Term 2023

November 21, 2023

Part I

A. STATISTICS

- **Numbers and percentages in each class.**

  See Table 1.

	Numbers						
	2023	2022	2021	2020	2019	2018	2017
Distinction	37	38	42	42	40	25	31
Merit	7	13	10	9	6	n/a	n/a
Pass	9	11	12	3	6	17	10
Fail	4	1	3	1	1	0	0
Total	57	63	67	55	53	42	41
	Percentages %						
	2023	2022	2021	2020	2019	2018	2017
Distinction	65	60	63	76	76	60	76
Merit	12	20	15	17	11	n/a	n/a
Pass	16	17	18	5	11	40	24
Fail	7	2	4	2	0	0	0
Total	100	100	100	100	100	100	100

Table 1: Numbers and percentages in each class

- **Numbers of vivas and effects of vivas on classes of result.**
  As in previous years there were no vivas conducted.
- **Marking of scripts.**

All dissertations and three mini-project subjects were double-marked. In cases of significant disagreement between marks, the two markers were consulted to agree a reconciled mark.

All written examinations and take-home exams were single-marked according to carefully checked model solutions and a pre-defined marking scheme, which was closely adhered to. A comprehensive independent checking procedure was followed.

B. New examining methods and procedures

Written examinations were all held in person this year in a closed-book format.

C. Changes in examining methods and procedures currently under discussion or contemplated for the future

None.

D. UCU Industrial Action

This academic year, the release of marks was delayed for 16 students due to the marking and assessment boycott (MAB). The Examiners initially awarded provisional degrees called 'Declared to Deserve Honours' and 'Declared to Deserve Master's' to 11 affected candidates for whom no more than one unit of assessment was missing. The remaining 5 affected candidates were not able to attend the summer graduation due to MAB-related incomplete results. Following the end of the boycott, an additional meeting of the examiners was held to finalise results for the affected candidates with complete academic records available. None of deemed deserved classifications were changed.

E. Notice of examination conventions for candidates

Notices to candidates were sent on: 17th October 2022 (first notice), 22nd November 2022 (second notice), 2nd March 2023 (third notice), 13th April 2023 (fourth notice) and 3rd May 2023 (final notice).

The examination conventions for the 2022-2023 academic year are online at http://mmathphys.physics.ox.ac.uk/students.

Part II

A. General Comments on the Examination

B. Equality and Diversity issues and breakdown of the results by gender

Table 2: Breakdown of results by gender

Class	Number																	
	2023			2022			2021			2020			2019			2018		
	F	M	Total	F	M	Total	F	M	Total	F	M	Total	F	M	Total	F	M	Total
D	5	32	37	2	36	38	3	39	42	7	35	42	5	35	40	1	24	25
M	3	4	7	3	10	13	3	7	10	3	9	9	1	5	6	n/a	n/a	n/a
P	3	6	9	6	5	11	4	8	12	0	0	3	0	6	6	3	14	17
F	1	3	4	0	1	1	0	3	3	1	0	1	1	0	1	0	0	0
Total	12	45	57	11	52	63	10	57	67	11	44	55	7	46	53	4	38	42
Class	Percentage																	
	2023			2022			2021			2020			2019			2018		
	F	M	Total	F	M	Total	F	M	Total	F	M	Total	F	M	Total	F	M	Total
D	42	71	65	18	70	60	30	69	63	64	80	76	72	76	75	25	63	60
M	25	9	12	27	19	20	30	12	15	27	20	11	14	11	n/a	n/a	n/a	n/a
P	25	13	16	55	10	17	40	14	8	0	0	11	0	13	11	75	37	40
F	8	7	7	0	2	2	0	5	4	9	0	2	14	0	2	0	0	0
Total	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100

Table 2 shows the performances of candidates broken down by gender.

C. Detailed numbers on candidates' performance in each part of the examination

The number of candidates taking each paper is shown in Table 3 and in the Average USM per Formal Assessment graph below. In accordance with University guidelines, statistics are not given for papers where the number of candidates was five or fewer.

Table 3: Statistics for individual papers

Paper	Number of Candidates	Avg USM	StDev USM
Advanced Fluid Dynamics	9	59	26
Advanced Philosophy of Physics	-	-	-
Advanced Quantum Field Theory	33	68	18
Advanced Quantum Theory	10	70	11
Algebraic Topology	-	-	-
Applied Complex Variables	6	57	17
Collisionless Plasma Physics	9	72	17
Dissertation (single unit)	12	72	-
Dissertation (double unit)	24	78	-
Galactic and Planetary Dynamics	7	70	6
General Relativity I	28	66	22
General Relativity II	16	61	23
Geophysical Fluid Dynamics	-	-	-
Groups and Representations	34	72	11
Introduction to Quantum Information	30	69	17
Kinetic Theory	13	66	11
Networks	7	74	5
Numerical Linear Algebra	9	50	20
Perturbation Methods	17	58	25
Quantum Field Theory	51	68	16
Quantum Matter	11	57	18
Radiative Processes and High Eng. Astro.	-	-	-
Random Matrix Theory	8	57	33
String Theory I	24	67	18
Supersymmetry and Supergravity	7	69	13

The number of candidates taking each homework-completion course is shown in Table 4. In accordance with University guidelines, statistics are not given for papers where the number of candidates was five or fewer.

Table 4: Numbers taking each homework completion course

Paper	Number of Candidates	Percentage completing course
Advanced Fluid Dynamics	-	-
Advanced Philosophy of Physics	-	-
Astroparticle Physics	9	100
Collisionless Plasma Physics	-	-
Collisional Plasma Physics	-	-
Conformal Field Theory	21	100
Cosmology	12	100
Galactic and Planetary Dynamics	-	-
Group and Representations	34	100
Kinetic Theory	-	-
Nonequilibrium Statistical Physics	15	100
Quantum Matter I	-	-
Quantum Matter II	8	100
Quantum Processes in Hot Plasma	-	-
Soft Matter Physics	7	100
String Theory II	10	100
Supersymmetry and Supergravity	12	100
Symbolic, Numerical and Graphical Scientific Programming	17	88
The Standard Model and Beyond I	7	86
The Standard Model and Beyond II	-	-
Topological Quantum Theory	26	100

D. Assessors' comments on sections and on individual questions

Advanced Fluid Dynamics

Question 1.

Part (a): The derivation of the dispersion relation was very standard and had been done in lectures. Most students did very well. A few students however did not seem to know that the divergence of the velocity is zero in an incompressible fluid.

Part (b): Surprisingly, many students could not give the correct expression of the magnetic or even kinetic energy.

Part (c): Not many students calculated the Poynting vector correctly and understood it had to be the energy per unit volume times the velocity at which the energy propagates.

Part (d): On the whole this questions was done correctly.

Question 2.

This question attracted a wide range of standards of answers, with one near-complete answer.

Part (a) was mostly done as expected, by observing that the curl operator commutes with $1 + \frac{1}{6}a^2\nabla^2$. The curl of $\mathbf{U} \cdot (\mathbf{I}/r + \mathbf{xx}/r^3)$ is a multiple of the harmonic function $\boldsymbol{\varphi}^{(1)}$ so the contribution from $\frac{1}{6}a^2\nabla^2$ acting on this vector field vanishes.

Part (b) should have been familiar (not least from the 2019 paper) but it was found the most difficult. Several candidates discarded the $\mathbf{xx}/r^3$ term, even though $\mathbf{xx}/r^3$ and $\mathbf{I}/r$ are both $O(1/r)$. The contribution from the strain rate then looks plausible, but has the wrong sign. The right way to exploit $r \gg a$ is to use $\nabla^2 \sim (1/r)^2$ acting on these expressions, so the $\frac{1}{6}a^2\nabla^2$ part of the velocity field is $O((a/r)^2)$ smaller than the leading-order part, and hence negligible for $r \gg a$. One then uses $\nabla \cdot \boldsymbol{\sigma} = 0$ to equate the integral of $\boldsymbol{\sigma} \cdot \mathbf{n}$ over a large sphere to the integral of $\boldsymbol{\sigma} \cdot \mathbf{n}$ over the surface $r = a$.

Part (c) was mostly done well, except almost all candidates forgot that the buoyancy force is proportional to the density difference $\widetilde{\rho} - \rho$, not to $\widetilde{\rho}$ alone.

Part (d) was mostly done well. A few candidates either ran out of time or did not spot the $s = \tan\phi$ substitution to evaluate the integral of $(1 + s^2)^{-3/2}$ with $s = Ut/\ell$ that arises from integrating the angular velocity of sphere B over time as sphere A falls down the z-axis. Sphere B rotates by angle $3a/2\ell$. The points on sphere B nearest the z-axis move downwards.

Advanced Quantum Field Theory

There was a minor and a more significant issue to note.

1. There was a typo in one of the formulae provided which was missing factors of 2π, which was noted by a few students. Both the provided version and the correct formula were accepted as correct answers.
2. There was a printing error in question 3, where part of the Feynman rules that were intended to be provided did not appear in the printed version of the paper. This made the question significantly harder and more time consuming.

The question by question breakdown is as follows:

Question 1. This question was based on scalar field theory and the foundational aspects of path integrals with some relatively simple calculation. The students generally did well on this question. Some students had difficulty with deriving the scaling of the correlation functions, diagrams and the S-matrix. Most students were comfortable with the diagrams and the loop integral. Some missed some diagrams that contribute (both at the connected level as well as to the amputated amplitude).

Question 2. This question was based on quantum electrodynamics. Students did quite well on this question. Many students had difficulty with part (e), which was asking about the dominant one-loop correction to electromagnetic scattering which arises from the vacuum polarization of the photon, and hence is universal.

Question 3. This was a question on non-Abelian gauge theories. The most difficulty students had was due to the above mentioned printing error. The question was graded without modifying the marking scheme. Partial credit was given where students made some progress towards the amplitude without the benefit of the explicit Feynman rules.

Advanced Quantum Theory

Question 1. This question concerns the classical statistical mechanics of one-dimensional lattice models, with the Q-state Potts model as the example. The methods are taught in detail in the lecture course but candidates were not expected to be familiar with this application. The question was answered excellently by a majority of candidates and well by the remainder. The most common mistakes were: a failure to picture excitations correctly in part (a) [some candidates viewed them as single minority spins rather than as pairs of domain walls, and so did not find the correct degeneracy of the first excited states]; and a failure to do a correct low-temperature expansion of the free energy in part (c).

Question 2. This question was about operator transformations applied to a model of a quantum spin chain. The general topic is taught in the lecture course but candidates were not expected to be familiar with the examples used in this question (the Schwinger boson representation of spin operators, and the spin-S XY ferromagnet). Parts (a) and (b) were answered well by most candidates but part (c) was done poorly by the majority. Some candidates had difficulties with the fact that the spin Hamiltonian is quartic in terms of Schwinger bosons rather than quadratic, and some candidates had difficulty in finding expectation values in a state (denoted $|G\rangle$ in the question) containing a non-zero boson density.

Collisionless Plasma Physics

Most students did fairly well in Q1. Part (a) was bookwork and so points were lost only for unclear/incomplete explanations and/or misreading the statement of the problem. For example, many students assumed that the definition of the conductivity tensor comes from the cold-plasma approximation. Part (b) was about ordering various plasma frequencies under a given set of assumptions; this proved to be an easy exercise. Part (c) proved to be slightly more challenging but for an unexpected reason — many students approached it via the Booker quartic, which makes the calculation several times harder than it needs to be. Additionally, several students thought that an additional assumption of $v_{\mathrm{th}} \ll c$

was needed, which is incorrect as this is a consequence of the orderings in part (b). Finally, everyone got reasonable final results for part (d), however, many lost points due to failing to recognise that speed of propagation of signals is the group velocity. I found this surprising given that I specifically pointed out the physics of whistler waves during the lectures and discussed group vs phase velocity.

Q2 was clearly more challenging, with only half of the students achieving a mark above 65 and one going below 50. Part (a) was standard bookwork that is explained in detail in Felix's lecture notes, and most showed good understanding of the WKB method. Same goes for the first part of (b), although several students failed to recognise that the derivation of the ray-tracing equations depends on $\nabla\mathbf{k}$ being a symmetric tensor, a consequence of $\mathbf{k} = \nabla S$. Part (c) wasn't an issue for the students. Part (d) was a very different story. First, note that there is an error in the statement of the problem: the correct expression for the RHS of the final result is $1 - d^2 \sin^2(\alpha)/r_0^2$. Fortunately, every student that got that far into the problem recognised the mistake and provided the correct final result. The solution to part (c) is very similar (even isomorphic) to various problems in orbital mechanics that should be familiar from any first-year mechanics course. Yet, several students failed at the very start of their attempt due to various issues with using cylindrical polar coordinates. This was very surprising.

The KMHD question (Q3) this year was done remarkably well by the majority of candidates. Since I am disinclined to revise my view that this was a fairly challenging question, I am logically forced to conclude that this was a very good cohort of students. Parts (a) and (b) required fairly standard algebra and were done more or less perfectly by everyone. In (b), only a few candidates failed to realise (or at least state clearly) that the result amounted to the statement that the rate of heating was minus work done by the pressure gradients on the plasma. Also, some took the long route (for which they were not penalised!) of calculating the rate of energy change via CGL equations rather than by directly multiplying the drift-kinetic equation in ε variables by ε and integrating (while being careful about how the Jacobian comes in). Part (c) was again mostly algebra, although not all of it completely straightforward, so a few students stumbled. The calculation of the rate of change of A was easier for those who had done the rate of change of K in (b) in the optimal way. Part (d) required quite a lot of nimble working, so again a few stumbled but the majority (just) acquitted themselves well. Some decided to assume a bi-Maxwellian distribution—an unnecessary (and, in general, incorrect) cop-out. Part (e) was an "understanding" question—results were a bit mixed, with a number of candidates going on irrelevant tangents, but most, all in all, having useful things to say.

Groups and Representations

The 34 student who sat this exam did quite well. The high average mark and students' actual work indicate that the main messages of the course are getting across. The more detailed break-down by question is as follows.

Question 1. A quite unpopular question only attempted by 8 students and with a relatively poor performance indicated by the average mark. Part (a), (b) and (c) were, on the whole, done quite well but students had significant problems with the application to field theories in parts (d) and (e).

Question 2. A question attempted by 29 students done quite well. The lecturer was actually somewhat concerned about this question, which is asking students to work out the representation theory of the permutation group S_4, a group of order 24, as some of the computations are a bit tricky. To his delight and the students' credit most went through this without significant problems. Part (e) was the one sub-question which sometimes caused difficulties.

Question 3. A question attempted by all students with a good mark average. Some problems arose in part (b) where a number of students were unable to apply Schur's Lemma correctly to the given example. Related problems affected part (d) where many students overlooked that the commutant contains a $U(1)$ factor.

Question 4. The question was attempted by 31 students with a good average. The first three parts were done very well and only part (d) caused major problems. Specifically, many students failed to state the correct embedding (or stated no embedding at all) which made it impossible to address the remainder of part (d). Where students did manage to write down the obvious embedding of $SU(2) \times SU(3)$ into $SU(2) \times SU(2) \times SU(4)$ they often struggled to identify the correct embedding of the hypercharge, $U_Y(1)$.

Kinetic Theory

Question 1. There was an unfortunate mistake in part (d), a missing factor of 1/2 on the right-hand side of the first equation. Full marks were given for either the answer as printed or the correct answer.

Part (a) was done well by almost all candidates.

Part (b) caused a lot of difficulty. The second result in part (a) establishes that the system does not conserve energy. The relevant form for the Liouville equation is the conservation form written using divergences with respect to $\mathbf{x}_i$ and $\mathbf{v}_i$,

$$\frac{\partial \varrho}{\partial t} + \sum_{i=1}^{N} \frac{\partial}{\partial \mathbf{x}_i} \cdot (\varrho \mathbf{v}_i) + \sum_{i=1}^{N} \frac{\partial}{\partial \mathbf{v}_i} \cdot \Big(\varrho \sum_{j=1}^{N} K(\mathbf{x}_i, \mathbf{x}_j)(\mathbf{v}_j - \mathbf{v}_i) \Big) = 0.$$

Almost all candidates wrote down an expression for a Hamiltonian system instead.

Part (c) was then made difficult because the Liouville equation was not written in conservation form, and so not convenient for integrating over $\mathbf{x}_2, \mathbf{v}_2, \ldots, \mathbf{x}_N, \mathbf{v}_N$ to obtain an evolution equation for the 1-particle marginal probability distribution ϱ_1. The contributions from $K(\mathbf{x}_j, \mathbf{x}_1)$ for $j = 2, \ldots, N$ can be replaced by $N - 1$ copies of the contribution from $K(\mathbf{x}_2, \mathbf{x}_1)$ as the particles are indistinguishable. The factor of $N-1$ is absorbed by setting $f_1 = N\varrho_1$ and $f_2 = N(N-1)\varrho_2$. We need to assume that the particles are uncorrelated to factorise the 2-particle distribution as

$$f_2(\mathbf{x}_1, \mathbf{v}_1, \mathbf{x}_2, \mathbf{v}_2, t) = f_1(\mathbf{x}_1, \mathbf{v}_1, t) f_1(\mathbf{x}_2, \mathbf{v}_2, t).$$

The calculations in part (d) were mostly done well, despite the mistake, a missing factor of 1/2 in the right-hand side of the first equation for the energy. Almost all candidates arrived at the energy equation as printed. A few obtained the correct equation. Both answers were given full marks.

Many candidates' calculations could have been shortened by starting with a suitable multiple of the kinetic equation ($\star$) with the $\mathbf{x}$-derivatives in divergence form:

$$\begin{aligned} \partial_t \left(\tfrac{1}{2} |\mathbf{v}|^2 f \right) + \nabla \cdot \left(\tfrac{1}{2}\, \mathbf{v} |\mathbf{v}|^2 f \right) + \tfrac{1}{2} |\mathbf{v}|^2 \nabla_{\mathbf{v}} \cdot (f \mathsf{L}[f]) &= 0, \\ \partial_t \left(f \log f \right) + \nabla \cdot \left(\mathbf{v} f \log f \right) + (1 + \log f) \nabla_{\mathbf{v}} \cdot (f \mathsf{L}[f]) &= 0, \end{aligned}$$

then integrating over $\mathbf{x}$ and integrating by parts in $\mathbf{v}$.

Very few candidates stated that this system obeys a *backwards* H-theorem. Boltzmann's function $H = \iint f \log f \, \mathrm{d}\mathbf{x} \, \mathrm{d}\mathbf{v}$ *increases*. The $K(\mathbf{x}_i, \mathbf{x}_j)$ interaction term reduces differences in velocities between particles, and hence increases correlations between particles.

Part (e) was mostly done well, though the calculations could often have been done more easily by integrating by parts in $\mathbf{v}$. The fluid density satisfies the usual mass conservation equation $\partial_t \rho + \nabla \cdot (\rho \mathbf{u}) = 0$.

Question 2. This question proved more challenging than anticipated. There were, however, students who acquitted themselves well, receiving marks in the 80% range and providing the proof of principle that this question *could* be done well. The majority who received low marks mostly did so due to astonishing inability to perform basic mathematical operations correctly, or at all.

Part (a) required inserting a Heaviside function into the standard expression for the dielectric function, which was provided in the script, and then doing the velocity integral. For the majority, the main difficulty was that the integral was over a 3D velocity space, whereas the Heaviside function was with respect to the absolute value of the velocity. The integral was easy to do if one went to polar coordinates in velocity space, but figuring that out proved beyond the capabilities of many students.

Part (b) was a qualitative question, requiring students to observe that, since the phase speed of the waves was larger than the Fermi speed, there were no particles in the distribution to resonate with the waves and hence no damping. Very few got that, although several still realised that there was no damping, by more tortuous, and in some cases partially incorrect, arguments.

Part (c) required Taylor-expanding the dispersion relation, given explicitly in the script, in small $kv_{\mathrm{F}e}/\omega$, and solving the resulting equation for the frequency. While a fair number understood that such an expansion was necessary, carrying it out correctly proved too daunting a task for most — although a few better educated souls did manage, and even realised that one had to go to higher order than perhaps initially anticipated in order to get the dispersive bit.

Part (d) dealt with the opposite limit, and the method for calculating the answer was explicitly stated in the script. Yet again, evaluating an elementary function approximately at small δ proved to be challenging for many.

Part (e) was again a qualitative exercise. Sketching the dispersion relation was hard to do right for those who did not calculate the Langmuir wave correctly. Students were generally aware that something to do with ions and sound waves might lurk at low frequencies, but clear and crisp thinking about what that was and what the coditions for its existence might be was in short supply.

Question 3. The students overall did rather well on this question, despite it being distinctly different from the questions of recent years (which had all been very similar to eachother). The marks ranged from 16/25 to 23/25.

Part (a) — all students got full marks.

Part (b) — this was done well, although it was frustrating how many times the students felt the need to ’ensemble average’, which suggests they were regurgitating the lecture notes. No such averaging was required or apposite here.

Part (c) — universally done rather well, with the odd slipped minus sign or factor of π.

Part (d) — this was largely done well although success in taking the limit was variable. Several students clearly fudged factors of $\pm\pi$, i etc in order to get the right answer, but they almost all had the right idea. Some were obviously running out of time by this stage.

Part (e) — much less well done. Most of the students could show that $\mathcal{T}$ was positive definite, earning them one mark, but few of them recognised that this was the torque on the *halo*, and therefore equal to minus the torque on the bar. Those who did realise this invariably understood that the bar would slow down, earning them the full three marks.

Quantum Field Theory

Question 1. This question was generally very well done. Some candidates confused loop order with coupling constant order, and a number thought that the addition of a ϕ^6 term to the Lagrangian made the theory non-renormalizable.

Question 2. This question caused most problems. Candidates tended to look for complicated internal symmetry groups; many could not derive the current or failed to appreciate how this could help with the rest of the question. Very few candidates were careful about the flow of charge in their Feynman rules, despite the wording of the question. Several weaker candidates simply assumed the question was the same as a class question in which one of the fields was fermionic.

Question 3. This question was on the whole well done. Some candidates do not understand that the Jacobian for an integration measure is an absolute value. A number were insecure on contracting covariant and contravariant indices - fortunately this didn't prevent them from working through the question.

Quantum Matter

The raw performance on this exam was not very good. Much of the material should have been fairly familiar though.

Question 1.

Parts (a) and (b) were done correctly by most. This material appears almost every year and should have been expected by most students. Part (c) was the hardest part of this question. Note, however, that parts (d) and (e) did not require one to correctly solve (c). Many students who got stuck on (c) did not even try (d) and (e) and thereby lost many marks.

On (c) only two students realized that one could use the fact that the single particle part of the Hamiltonian has eigenenergies E_0 and E_1. This was not required, but would have made the calculation much simpler (perhaps one should have noticed that these variables were defined in the question and then you never used them!). Quite a few students managed to solve this part of the problem without using E_0 and E_1, although a lot of errors occurred on the way.

Parts (d) and (e) were done poorly by most, with only one student getting full credit on both parts. This was not meant to be difficult. One writes a coherent state

$$\exp\left[\sqrt{N}(\cos\theta a_L^\dagger + \sin\theta e^{i\chi} a_R^\dagger)\right]|0\rangle$$

since a_L and a_R commute, you can write this as the product of two coherent states

$$\exp\left[\sqrt{N}\cos\theta a_L^\dagger\right]|0\rangle_L \times \exp\left[\sqrt{N}\sin\theta e^{i\chi} a_R^\dagger\right]|0\rangle_R$$

so that one just takes H as given and replaces $a_L \to \sqrt{N}\cos\theta$ and $a_R \to \sqrt{N}\sin\theta e^{i\chi}$.

Question 2.

This question seemed to catch students off-guard. Admittedly Fermi-Liquid theory had never shown up on exams before. However, the question is essentially identical to a homework question, except on the exam it is in 2D rather than 3D. This should have made the question easier rather than harder!

Part (a) and (c) were easy and were done correctly by most.

Part (b) was supposed to be easy marks. Density of states in 2D is a standard question in both the 2nd year and 3rd year syllabus, and yet most students couldn't do it for some reason. A remarkably large fraction of students gave answers that didn't even have sensible dimensions.

For part (d) many students understood that this is basically a Taylor expansion. For full marks the student would have to explain that quasiparticles become good eigenstates, or that scattering doesnt' happen or something of the sort (and only a few students did that).

Part (e) was the main body of the problem and was done very badly (high score 6/10). About half of the students made a decent start to the calculation with a clear strategy, but none managed to push the calculation through (as in the homework problem). The first term of the free energy should have just given the same result as (a) but with effective mass. Only one or two students managed to get this far. The interaction term was the hard part and no one managed to do it fully.

Radiative Processes and High Energy Astrophysics

Question 1. all students were able to answer this question relatively well. The most important difficulties encountered were in deriving the additional factor of 1/2 in part (c), in explaining the origin of the Q factor in part (f), and in deriving its functional form in part (g).

Question 2. all students were able to answer this question quite well. The most common difficulty was justifying why one could ignore stimulated emission in part (b).

Overall comment: some of the students failed to properly define some key quantities (e.g. $\langle \sigma v \rangle$ was defined a the probability of annihilation per unit time, and the Einstein coefficients $B_{12/21}$ were defined as the probabilities of absorption/stimulated emission per unit time, without accounting for the incident radiation density).

Supersymmetry and Supergravity

The number of candidates who opted for the exam was relatively small (7). The answers in the first part were at a high level of accuracy with mostly minor mistakes or omissions. One candidate did not make a full attempt at answering (b) and another got derailed in (c). No particular common difficulties or misconceptions stood out to me. The answers in the second part showed a good level of understanding as well. In (b) a few students derived the $\{Q, \bar{Q}\}$ anticommutators correctly but omitted $\{Q, Q\}$ and $\{\bar{Q}, \bar{Q}\}$, leading to minor deductions. In d) the identification of the R-charges was a somewhat common difficulty.

C3.1: Algebraic Topology

Question 1. Many candidates took an explicit simplical approach to part (a.i), though a few did quote Kunneth. Part (a.ii) was generally fine, though with some confusions about the generating 1-cocycles. Parts (b) and (c.i) were fine for the most part. The final part (c.ii) saw many reasonable answers, though there was variation in level of thoroughness and what elements were taken as clear.

Question 2. Most serious answers were alright for parts (a) and (c), and also for part (b) though with some differences of opinion about what was too obvious to require mentioning. Though some candidates did not get part (d.iii), a fair number did work through to a correct answer to this more difficult part.

Question 3. Most answers to part (a) were reasonable, though with some errors in low degrees. In part (b) some candidates reduced the question to that of the degree of a reflection on the sphere but either presumed the the answer there (which amounted to presuming the core of the question) or did not sufficiently justify that degree. Most answers to part (c) were reasonable, though with some issues about the quantifications in the definition. Though there were some correct answers to part (d), also a number of candidates tried (mostly with errors) to use a global homological characterization of orientability, expressly against the indication to use the local notion from part (c).

C3.12: Low-Dimensional Topology and Knot Theory

Solutions for Question 1 were generally good, with some candidates failing to require cobordisms to be compact in 1(a)(i). Solutions for 1(a)(ii) were essentially all correct. In 1(a)(iii), several candidates failed to check inverses. In (b)(i), candidates usually had the right idea. Part (b)(ii) proved to be more difficult, but there were several different correct approaches among the solutions, including doubling and the long exact sequence of a pair. Solutions for (b)(iii) were typically correct. There were essentially no complete solutions for part (c), but many partial results. Showing that the connected sum of an even number of copies of the projective plane is null-cobordant was usually missing.

Overall, there were lots of good solutions for Question 2. In part (a), some people forgot to require a Seifert surface to be compact. In part (b), many candidates got the wrong Seifert matrix, due to miscalculating linking numbers. This did not affect the marks given for (b)(iii) and (c). Most solutions for (c) were correct.

There was just one solution for Question 3, which was essentially correct.

C5.5: Perturbation Methods

Overall Question 1 was popular. The first part of the question and the path of steepest descent was generally tackled very well. For the next part of the question, the choice of the appropriate contour for the use of the steepest descent method proved to be a genuine hurdle in the question for a number of candidates, while only the best attempts successfully expanded about the location of the dominant contribution to the steepest descent integral. Many candidates parametrised the steepest descent curve with respect to ζ, without properly accounting for the fact the steepest descent curve has an infinite gradient with respect to ζ at the location of the dominant contribution or recognising that an alternative parametrisation may have been more convenient.

Question 2 appeared to be the least favourite question. A few attempts gathered difficulties early and these candidates generally moved onto the other questions, while there was also a number of very high scoring solutions. Many candidates did not apply the method of intermediate variable matching correctly, while the most successful solutions recognised which terms had to balance when matching via the intermediate variable method.

Question 3 started with bookwork concerning important definitions that candidates knew well. An occasional candidate used a different method than requested in part (b), and a few candidates struggled, but on the whole part (b) was executed very well. The final part differentiated most attempts. In particular keeping track of the level of approximation and the terms that need to cancel between the two integral contributions in the use of the domain splitting method to more than leading order typically, but not always, proved problematic for the candidates.

C5.6: Applied Complex Variables

Question 1. The part which caused the most trouble on this question was part (a), perhaps because it was a little unusual. The rest of the question was handled very well on the whole. Some candidates made mistakes identifying the velocity and the potential at the point A, as the sink is approached.

Question 2. Most of this question was handled very well. In part (c) a common mistake was to inadvertently give a definition for $\frac{(z-1)^{1/2}}{z^{1/2}}$ rather than $\frac{(1-z)^{1/2}}{z^{1/2}}$. No candidate managed to get the answer out in part (d), which required care with branches of square roots when evaluating the extra pole contributions. However, a good many managed to solve part (e), given the answer which had been provided in (d). There was one small mistake in Q2(e) which should have asked candidates to find a solution bounded at infinity (the solution is unbounded at the origin).

Question 3. This was a very popular question and there were a lot of very good answers. The bookwork was handled well, and very few candidates had trouble despite the additive decomposition having a double pole.

C6.1: Numerical Linear Algebra

Q2 was the most popular, and was attempted by over 80% of the candidates. Q1 was attempted by slightly fewer candidates than Q3.

Q1: some struggled to use the Courant-Fischer theorem properly in a(i) to get the desired inequality. a(iv) was a new problem requiring some guessing and computation, and seemed to be very challenging. In (b) some failed to note the assumption $k(A) \gg 1$; when $k(A) = O(1)$ it is easy to come up with examples, but the inequality is not very interesting.

Q2(b): while most correctly used the connection between the power method and QR algorithm to discuss the convergence of the latter, very few noted the requirement in the power method convergence that the initial vector has nonzero components in the the dominant eigenvector. Q2(c)(ii): some presented examples that are triangular or diagonal; while the QR algorithm may not change these matrices much (or at all), this is not a good example as such matrices have already converged! (d)(i) appears to have been very challenging. One needs to use the backward stability of QR factorisation and orthogonal matrix multiplication to prove one step of QR algorithm is backward stable.

Q3(a) (i,ii): A fair number of candidates wrote H^{-1}; this is inappropriate as H is not even square. Some answered (iii)(c) by noting that once the exact solution is found GMRES stops making progress; this is technically correct (and received marks) but the intended solution was to note that GMRES can stagnate even before the solution is found; a fact indicated in a question in the problem sheets. (b) appears to have been challenging, even though it is pretty similar to the discussion in lectures and a question in problem sheets. Most attempts failed to use the orthogonal invariance of Gaussian matrices together with a QR (or SVD) of A.

C7.4: Introduction to Quantum Information

Question 1. It was by far the most popular question, attempted by *all* of the students. Perhaps not so surprising, given that the question was based on the mainstream material. The book-work in part (a) was very well answered. In part (b) the students showed a good grasp of the Born rule but many of them struggled with calculations that led to the $\mathrm{Tr}(U)$ expression. Part (c), again, was almost perfectly answered, most likely because the question did not explicitly ask for a detailed calculation of the posterior probabilities. Those few who attempted to calculate the posterior probabilities made minor mistakes, even though they managed to arrive at the right conclusion. Part (d) turned out to be the most difficult one, with many students failing to use the fact that the eigenvalues of a unitary matrix are of the form $e^{i}\theta$. Instead many attempted to obtain the eigenvalues from the constraints on the trace and the determinant. This is a good alternative approach, but most of the students who took this route could not see the relevance of the *real* part of $\mathrm{Tr}(U)$ when deriving the probability from which the eigenvalues of U are then obtained.

Question 2. In general, the question was well answered and students scored well. Part (a) was book-work but, surprisingly, many students couldn't succinctly justify the answers; part (b) was done in a few different ways, but almost always successfully; in parts (c) and (d) most students dropped a few marks, having struggled with upper bounds; part (e) was usually answered correctly using mathematical induction; part (f) was unproblematic and very few students got it wrong (usually silly mistakes).

Question 3. At first glance this question might have looked difficult for it contained new topics (encryption of quantum states), hence it was not very popular, but those who attempted it did quite well. Part (a) was similar to one of the class problems and most students provided correct answers, but only few supplemented it with geometric interpretation. Students knew how to handle parts (b) and (c) but most did it by analysing specific cases, rather than using general notation. Showing that compositions of Clifford gates are Clifford gates in part (d) posed no problems. Most students noticed that part (e) is a generalisation of part (c) and provided a reasonable description of delegated quantum computation based on Clifford gates. Part (f) was well answered but hardly anyone commented on the need to go beyond the Clifford gates.

C7.5: General Relativity I

Question 1. This question was very popular and attempted by most students. The majority were able to do parts *a*-*c* without too much difficulty, although a surprising number of students assumed that the curve γ was a geodesic, despite the question explicitly saying that this may not be the case. Those students who struggled with parts *b* and *c* also often seemed to be under the impression that all curves are geodesics. Part *d* required some more algebra, and the ability to convert between abstract tensor expressions and concrete expressions for derivatives of functions along curves – this proved a challenge to a number of students. A frequent error here was believing that the t derivative of the t-component of a vector is always 1, while in fact, in this question, the t-component of the vector in question is a constant (and so its t derivative vanishes). Finally, part *e* should really have been approached as a system of linear ODEs, but almost no students did this. Instead, the majority of students who attempted part *e* derived a second order ODE for one component of Y, and in doing so showed that this component undergoes periodic oscillation – though they rarely went on to show that the other components also oscillate periodically. Overall, most students scored well in the parts of the question they attempted, and low scoring students most often offered partial answers to only a few parts of the questions (the "bookwork" parts) and spent time copying out parts of the question, while leaving other parts of the question completely untouched.

Question 2. This was by far the least popular question, and was only attempted by a handful of students, probably because it was the least familiar in style (compared with past exam questions). Most of the students who did attempt it did well, however, scoring slightly higher on average than the other two questions. Part *a* was not completely straightforward but almost all students were able to do it well, and part *b* required an understanding of normal coordinates and special relativity which was also demonstrated by almost all students. Part *c* was the most difficult part of the question, requiring some fairly intricate algebraic manipulation, and in fact no student was able to completely solve this part of the question, though some came very close. Part *d* was generally done fairly well, even by those students who could not complete part *c*, although no student made explicit the crucial fact that the coordinate vector fields are parallel-transported in Minkowski space.

Question 3. This was a very popular question, with the vast majority of students attempting it together with question 1. Part *a* was done very successfully by almost all students, with only a small minority forgetting that the Lagrangian itself is a conserved quantity (when the curve is parametrised by an affine parameter). Part *b*, however, was generally not done successfully – in fact, no student completely solved this part of the question, though some came very close. A very common error was to assume that a geodesic which is *emitted* radially will always remain radial, whereas in fact (since the spacetime is rotating) the geodesic will itself start to rotate. The key point was to realise that the conserved angular momentum is zero: noticing this made the rest of the algebra considerably easier. Even taking this fact into consideration, the resulting integral was not accurately solved by any student – the easiest way to solve it is to first make a substitution of variables to remove the hyperbolic cosine, and then to remember the formulae for derivatives of inverse trig functions, and while some students were able to perform one of these operations, no student did both. In retrospect this integral is probably too difficult without a hint. Finally, students generally faired better on part *c*, although a surprisingly large number of students made algebraic mistakes in solving the quadratic inequality in part *c (i)* (perhaps they were running out of time when trying this question), and some students made the common mistake of believing that every curve is a geodesic. Finally, most answers to part *c (ii)* were nonsense, and while some students said something true but trivial (e.g. that there are timelike circular orbits only in the interior region – although even this statement is true only if "circular" is interpreted in a coordinate-relative manner), only one student identified the closed timelike curves.

C7.6: General Relativity II

Question 1. This problem exploring the redshift formula, the stress-energy and the Ricci tensors in the (unnamed) Janis-Newman-Winicour metric was attempted by the majority of the candidates. The typical issues were the following.

- Confusing the coordinate time with the proper time, namely taking the velocity vector of an observer following a curve $\gamma^\mu = (t, r_0, \theta_0, \phi_0)$ with constant r_0, θ_0, ϕ_0 to be simply (1, 0, 0, 0), which is off by a factor of $(g_{tt}|_\gamma)^{-1/2}$.
- Taking the wave vector of a null ray $\gamma^\mu(\lambda)$ to be the velocity vector, as opposed to $\dot{\gamma}^\mu$, where k is the wave number.
- Ignoring or not taking full advantage of the trace reversal in the Einstein field equations.

Judging by the candidates' performance, this problem may have been the most challenging.

Question 2. This problem on Einstein's quadrupole formula was only tackled by one candidate - and with a very decent level of success. The unpopularity of the problem may indicate the propensity of the students attempting the exam towards more typical problems involving exact metrics.

Qustion 3. This problem exploring the Killing horizon of the (unnamed) extremal Kerr solution was attempted by all candidates. The typical mistakes were the following.

- In the context of a hypersurface $\sum$ defined by $r = \text{Const}$, identifying normal vector N with ∂_r instead of taking the normal covector to be $n \propto dr$, as implied by the regular-value theorem.
- Having observed the normal vector to be $N = aT|_\sum + bL|_\sum =: K|_\sum$, where both T and L are Killing vector fields, extending the Killing vector field K away from $\sum$ with non-constant a and b. This reflects the complexity of the concept of Killing horizon, which relies on the non-trivial combination of vector fields defined on and away from it.

C7.7: Random Matrix Theory

Question 1 was attempted by most of the candidates. Parts (a), (b) and (c) were straightforward and were in general answered well. Most candidates found part (d) difficult. Only a few calculated the variance, as asked for; many only established an order estimate for it, but could then still prove almost sure convergence successfully. Only a few candidates correctly identified the paths that give a non-zero contrition in the limit and that the contributions from these can be evaluated using the information given in the question.

Question 2 was attempted by roughly half the candidates. Parts (a), (b) and

(d) were straightforward. In answering part (b), some candidates failed to say that the random variables need to be paired with their complex conjugates. Many candidates did well on part (c)(i), but some attempted a more general calculation than was asked for. Part (c)(ii) was challenging and only a few candidates scored well on it. Many didn't see that the permutations fall into two classes, with permutations in each class giving the same contribution. Part (e) was also challenging and only a few candidates scored well on it. Many failed to take advantage of the fact that the matrix entries were stated to be Gaussian random variables and so to use Wick's theorem, despite the question saying to do this.

Question 3 was attempted by roughly half the candidates. Most of those who did attempt it gained high marks. Part (a) was straightforward. Many candidates saw that using the Fourier expansion for the ratio of sine functions considerably simplifies the calculation, but not all did. Most found parts (b) and (c) straightforward too, although some failed to apply Gaudin's Lemma correctly. Several candidates did part (d) well, but some failed to spot the connection to the two-point correlation function, which simplifies the calculation considerably.

E. Comments on performance of identifiable individuals

Prizes

Prizes were awarded to the following candidates:

The top prize was awarded to:

Qi Huang (St Hugh's College)

Prizes were also awarded to:

Ylias Sadki (Somerville College)
Henry Stubbs (Worcester College)

One student was highly commended for his dissertation:

Kieran Cooney (Oriel College)

Mitigating Circumstances Notices to Examiners

The Examiners received 5 applications regarding mitigating circumstances. The Examiners considered the applications carefully and agreed appropriate action.

F. Names of members of the Board of Examiners

Examiners:

Prof Alex Schekochihin (Chair, Department of Physics, University of Oxford)
Prof Christopher Beem (Mathematical Institute, University Of Oxford)
Prof Artur Ekert (Mathematical Institute, University Of Oxford)
Prof John Magorrian (Department of Physics, University of Oxford)
Prof Martin Evans (School of Physics and Astronomy, University of Edinbrugh)
Prof Toby Wiseman (Blacklett Laboratory, Imperial College London)

Assessors:
Dr Abhishodh Prakash
Dr Adam Caulton
Prof Alex Schekochihin
Prof Andras Juhasz
Prof Andre Henriques
Prof Andre Lukas
Prof Andrei Constantin
Prof Andrew Wells
Prof Ard Louis
Prof Aris Karastergiou
Prof Artur Ekert
Prof Bence Kocsis
Dr Carolin Wille
Prof Caroline Terquem
Dr Chiara Marietto
Dr Christopher Couzens
Dr Chris Hamilton
Prof Chris Hays
Dr Christoph Uhlemann
Prof Christopher Beem
Prof Cornelia Drutu
Prof Damian Rossler
Dr David Alonso
Prof Dominic Joyce
Prof Fabian Essler
Prof Fabrizio Caola
Prof Fernando Alday
Dr Gabriel Machuca

Prof Gavin Salam
Dr Gowri Kurup
Dr Jack Helliwell
Prof James Binney
Prof James Sparks
Dr James Wills
Dr Jan Rozman
Prof Jason Lotay
Dr Jingxiang Wu
Prof John Chalker
Prof John Magorrian
Prof John March-Russell
Prof John Wheater
Prof Jon Chapman
Prof Jon Keating
Prof Jonathan Barret
Prof Joseph Conlon
Dr Josu Aurrekoetxea
Prof Julia Yeomans
Dr Lakshya Bhardwaj
Dr Ling Lin
Dr Mario Reig
Prof Mark Mezei
Dr Nayara Fonseca
Dr Nick Bultinck
Dr Nicholas Ormrod
Dr Nicola Pedreschi
Dr Nora Martin
Prof Paul Dellar
Prof Pedro Ferreira
Prof Peter Grindrod
Prof Peter Norreys
Prof Philip Candelas
Dr Pieter Bomans
Dr Plamen Ivanov
Prof Prateek Agrawal
Dr Ramy Aboushelbaya
Prof Ramin Golestanian
Prof Renaud Lamboitte
Prof Ruth Baker

Prof Sakura Schafer-Nameki
Dr Sarah Newton
Dr Sebastian von Hausegger
Prof Shivaji Sondhi
Prof Siddharth Parameswaran
Prof Steven Balbus
Prof Steven Simon
Prof Tim Wollings
Prof Yuji Nakatsukasa