Introduction to Topological Phases of Matter Mini-Project

Master of Science in Mathematical and Theoretical Physics

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In this mini-project, you will investigate the two-dimensional Z_2 topological insulator — a time-reversal invariant phase of matter characterized by robust edge states and a gapped bulk. This system exemplifies a symmetry-protected topological (SPT) phase and plays a central role in the study of topological band theory.

This project is part of a course on topological ideas in condensed matter, and your report should reflect a serious engagement with the mathematical and conceptual underpinnings of the phase, not just its physical phenomenology.

Project Requirements

You are expected to:

- Provide a clear and rigorous description of what the Z_2 topological insulator is, including a discussion of the relevant symmetries and topological invariants (especially the Z_2 index).
- Explain the topological ideas involved, including concepts such as the Berry connection, time-reversal symmetry, and the bulk-boundary correspondence. A conceptual comparison with the Chern insulator may be helpful.
- Analyze a concrete lattice model for the Z_2 topological insulator (e.g., the Bernevig–Hughes–Zhang (BHZ) model or equivalent). You must:
  - Implement the model numerically on both a torus and a cylinder .
  - Compute and visualize relevant observables (e.g., band structure, edge states, topological invariants).
  - Interpret your results in the context of topological classification and the role of boundaries.
  - Show that there is a transition to a topologically trivial phase as the parameters in the model are varied
- Discuss how the model connects to experimental systems (e.g., HgTe/CdTe quantum wells) and what features are observable in practice.

Suggested References

There is a basic overview in our core textbook: Moessner and Moore, Chapter 3.
The following additional selected references provide theoretical foundations, model analysis, and historical context. You are not expected to cover all of them in depth, but they offer useful entry points:

- R. Roy, *Topological phases and the quantum spin Hall effect in three dimensions*, Phys. Rev. B **79**, 195321 (2009). DOI: 10.1103/PhysRevB.79.195321. A lesser known but very early paper introducing the Z_2 *topological invariant* from the perspective of *band topology* and time-reversal symmetry.
- J. K. Asbóth, L. Oroszlány, and A. Pályi, *A Short Course on Topological Insulators*, Springer (2016). Available at: `https://arxiv.org/abs/1509.02295`. Pedagogical and computationally oriented.
- M. Z. Hasan and C. L. Kane, "Colloquium: Topological Insulators," *Rev. Mod. Phys.* **82**, 3045 (2010). DOI: 10.1103/RevModPhys.82.3045. Comprehensive review of the theory and experiments.
- B. A. Bernevig and T. L. Hughes, *Topological Insulators and Topological Superconductors*, Princeton University Press (2013). A detailed textbook treatment, including the BHZ model.

Formatting Guidelines

- Length: 8-10 pages (2 pages max with figures) + references + code
- Font size: 12 point (single spaced)
- Margins: One inch on all sides

You may assume standard graduate-level background in quantum mechanics and band theory. Be explicit in your discussion of which ideas come from existing sources, and cite them properly. Original research is not required, but you must produce and analyze your own numerical data.