**ADVANCED PHILOSOPHY OF PHYSICS**

MICHAELMAS TERM

**Lecturers:** [Prof Adam Caulton](https://www.philosophy.ox.ac.uk/people/adam-caulton), [Prof James Read](https://www.philosophy.ox.ac.uk/people/james-read), [Prof Christopher Timpson](https://www.philosophy.ox.ac.uk/people/chris-timpson)

**Course Term:** Michaelmas

**Course Weight:** 1.5 units/24 lectures

**Assessment Method:** mini-project **or** homework completion

**Course Synopsis:**

This series of classes will cover contemporary topics in the philosophy of physics, with emphasis on: thermal physics (thermodynamics and statistical mechanics), the role of symmetries in physical theories, spacetime (especially the general theory of relativity), and advanced topics in the philosophy of quantum theory (which may include the role of decoherence in solving the measurement problem, the interpretation of probability, and topics in quantum field theory).

Those MMathPhys and MSc students taking the course in the mini-project or homework mode also receive 8 hours of tutorials (usually as one of a pair) from one of the external lecturers. These tutorials are usually spread throughout the year, with the first 4 in Michaelmas Term. Students are expected to produce an essay of between 2,000-2,500 words for each tutorial. For those taking the course in mini-project mode, two of these tutorials will be given over to discussing drafts of the two 5,000-word essays submitted by each candidate for assessment in week 4 of Trinity Term.

**ANYONS AND TOPOLOGOCIAL QUANTUM FIELD THEORY**

**Lecturer:** [Prof Steve Simon](https://www-thphys.physics.ox.ac.uk/people/SteveSimon/)

**Course Term:** Michaelmas

**Course Weight:** 1 unit/ 16 lectures

**Assessment Method:** written exam in HT week 0 **or** homework completion

**Course Synopsis:**

Lecture 1

* What this course is about, and how we will run it
* Historical digression about topology in quantum physics
* Knots and Knot Invariants
* The Kaufmann Invariant
* Relation to Quantum Physics and Topological Quantum Theory
* Connection to Condensed Matter Experiments
* Twists, Spin, Statistics and the Jones Polynomial
* Relation to Quantum Information / Quantum Computation
* Computational Power of Topological Quantum Systems
* Brief Description of Topological Quantum Computers
* Brief Description of Connection Between Knots and Quantum Mechanics
* Rough Picture of Fractional Quantum Hall Effect and Why it is Topological

Lecture 2

* + Particle Quantum Statistics
  + Beyond Bosons and Fermions
  + Digression on Path Integrals
  + Topology of Paths in 2+1 and 3+1 Dimensions
  + Digression on Group Theory and Representations
  + Composition of Paths
  + Permutation Group and Braid Group
  + Back to Path Integrals
  + Fractional Statistics / Anyons
  + Non - Abelian Anyons
  + Relation to Quantum Computation

Lecture 3

* + Aharonov - Bohm Effect; a useful Application of Path Integrals
  + Understanding this in Path Integral Language
  + Flux - Charge Composites and Fractional Statistics
  + Spin of Anyons / Spin - Statistics
  + Fusion
  + Ground State Degeneracy and Topological Quantum Memory

Lecture 4

* + Abelian Chern - Simons Theory and Flux Binding
  + Manifold Partition Function / Ground State Degeneracy
  + NonAbelian Chern-Simons Theory
  + Wilson Loops and Knot Invariants

Lecture 5

* + Connections to Quantum Gravity

Lecture 6

* + Defining a Topological Quantum Field Theory
  + Atiyah Axioms
  + Hilbert Spaces,
  + Gluing Space - Time Manifolds
  + Manifolds with Particles
  + Building Manifolds by Gluing
  + Overlaps and the S - matrix

Lecture 7

* + Structure of Topological Quantum Field Theory
  + Structure of Hilbert Space
  + Fusion of Nonabelian Anyons
  + Quantum Dimension
  + Importance of Locality
  + Examples:
    - Fibonacci Anyons
    - Ising Anyons
  + Ground State Degeneracy and Fusion Rules

Lecture 8

* + Beginning to Define a Full Anyon Theory / TQFT / Fusion Category
  + F-matrix and Pentagon Equation
  + Braiding
  + Fibonacci Example
  + Hexagon Equation
  + Topological Quantum Computing with Braids

Lecture 9

o Diagram Algebra

o Meaning of Diagams

o Isotopy Invariance

o Twists

o Ribbon

o S-matrix and Verlinde Formula

Lecture 10

o Manifold Surgery

o Likorish - Wallace Theorem and Kirby Calculus

o Witten - Reshitikhin - Turaev Invariant

Lecture 11

o Basics of Quantum Information – Qubits

* Codes and Errors – Classical Case
* No Cloning Theorem

Lecture 12

o Kitaev's Toric Code / Surface Code

 Commuting Projection Operators

 Protected Code Space

 Checking and Correcting Errors

 String Operators and Handles

o Toric Code as a Phase of Matter

o Braiding Properties

o Twists and S - matrix

o Robustness of the Phase

o Defining Topologically Ordered Matter

Lecture 13

o Generalizing the Toric Code: The Kitaev Model

o Lattice Gauge Theory

Lecture 14

o Generalizing Toric Code

o Toric Code as Abstract Quantum Loop Gas

 Algebra of Loops

 Quasiparticles

o Double Semion Loop Gas

 Algebra of Loops

 Quasiparticles

o Doubled Fibonacci String Net

o Levin - Wen Model

**FIELD THEORIES AND COLLECTIVE PHENOMENA IN CONDENSED MATTER**

**Lecturer:** [Prof John Chalker](https://www.physics.ox.ac.uk/our-people/chalker)

**Course Term:** Michaelmas

**Course Weight:** 1.5 units/ 24 lectures

**Assessment Method:** written exam in TT

**Course Synopsis:** Path integrals in Quantum Mechanics; the propagator. Path Integrals in Quantum Statistical Mechanics; correlation functions; perturbation theory; Feynman diagrams. Path Integrals and Transfer Matrices. Transfer matrix approach to the Ising Model. Second quantisation. Ideal Fermi gas in second quantization. Weakly interacting Bose gas: Bogoliubov theory; superfluidity. Spinwaves in a ferromagnet. Landau theory of phase transitions. Stochastic processes and path integrals; Brownian motion and the Langevin equation.

**GROUPS AND REPRESENTATIONS**

**Lecturer:** [Prof Andre Lukas](https://www.physics.ox.ac.uk/our-people/lukas)

**Course Term:** Michaelmas

**Course Weight:** 1.5 units/ 24 lectures

**Assessment Method:** written exam in HT week 0 **and** homework completion

**Course Synopsis:**

Basics on groups, representations, Schur's Lemma, representations of infinite groups, Lie groups, Lie algebras, Lorentz and Poincare groups, spinor representations, roots, classification of simple Lie algebras, weights, representations and Dynkin formalism.

**KINETIC THEORY**

**Lecturers:** [Prof Alex Schekochihin](https://www.physics.ox.ac.uk/our-people/schekochihin), [Dr Paul Dellar](https://www.maths.ox.ac.uk/people/paul.dellar)and[Dr Chris Hamilton](https://www.physics.ox.ac.uk/our-people/hamiltonc)

**Course Term:** Michaelmas

**Course Weight:** 1.75 units/28 lectures

**Assessment Method:** written exam in week 0 HT **or** homework completion

**Areas:** CMT, Astro, foundational course

**Sequel:** Advanced Fluid Dynamics (HT), Collisionless Plasma Physics (TT), Collisional Plasma Physics(TT), Galactic and Planetary Dynamics (HT)

**Course Synopsis:**

Part I (9 lectures). Kinetic theory of gases. Timescales and length scales. Hamiltonian mechanics of N particles. Liouville’s Theorem. Reduced distributions. BBGKY hierarchy. Boltzmann-Grad limit and truncation of BBGKY equation for the 2-particle distribution assuming a short-range potential. Boltzmann's collision operator and its conservation properties. Boltzmann's entropy and the H-theorem. Maxwell-Boltzmann distribution. Linearised collision operator. Model collision operators: the BGK operator, Fokker-Planck operator. Derivation of hydrodynamics via Chapman-Enskog expansion. Viscosity and thermal conductivity.

Part II (10 lectures). Kinetic theory of plasmas and quasiparticles. Kinetic description of a plasma: Debye shielding, micro- vs. macroscopic fields, Vlasov-Maxwell equations. Klimontovich’s version of BBGKY (non-examinable). Plasma frequency. Partition of the dynamics into equilibrium and fluctuations. Linear theory: initial-value problem for the Vlasov-Poisson system, Laplace-tranform solution, the dielectric function, Landau prescription for calculating velocity integrals, Langmuir waves, Landau damping and kinetic instabilities (driven by beams, streams and bumps on tail), Weibel instability (non-examinable), sound waves, their damping, ion-acoustic instability, ion-Langmuir oscillations. Energy conservation. Heating. Entropy and free energy. Ballistic response and phase mixing. Role of collisions. Elements of kinetic stability theory. Quasilinear theory: general scheme. QLT for bump-on-tail instability in 1D. Introduction to quasiparticle kinetics.

Part III (9 lectures). Kinetic theory of self-gravitating systems. Unshielded nature of gravity and implications for self-gravitating systems. Virial theorem, negative specific heat and impossibility of thermal equilibrium. Escape, impact of fluctuations. Mean-field approximation, angle-action variables, self-consistent potential, biorthonormal potential-density pairs. Relaxation driven by fluctuations in mean-field. Long-time response to initial perturbation. Fokker-Planck equation. Computation of the diffusion coefficients in terms of resonant interactions. Application to a tepid disc.

**QUANTUM FIELD THEORY**

**Lecturer:** [Prof John Wheater](https://www.physics.ox.ac.uk/our-people/wheater)

**Course Term:** Michaelmas

**Course Weight:** 1.5 units/24 lectures

**Assessment Method:** written exam in week 0 HT

**General Prerequisites:** The course is on QFT, not on its precursors. To help you make sure that you are prepared there will be some preliminary material available on the Course Materials tab.

**Areas:** PT, CMT, Astro, foundational course

**Sequels:** Advanced Quantum Field Theory for Particle Physics (HT), Conformal Field Theory (TT), Quantum Field Theory in Curved Space-Time (TT)

**Course Synopsis:**

1. Introduction, and Why do we need quantum field theory?  
2. Relativistic wave equations  
3. Formalism of classical field theory  
4. Canonical quantisation of the real scalar field  
5. Charge and complex fields  
6. Canonical quantisation of the fermion field  
7. Interacting fields, formalism and the perturbation expansion  
8. Scattering and decay, their relation to amplitudes  
9. Calculation of low order Feynman diagrams  
10. Regularization, effective and renormalizable QFTs  
11. Feynman path integral quantisation

**QUANTUM PROCESSES IN HOT PLASMA**

**Lecturer:** [Prof Peter Norreys](https://www.physics.ox.ac.uk/our-people/norreys)

**Course Term:** Michaelmas

**Course Weight:** 0.75 units/12 lectures

**Assessment Method:** homework completion only

**General Prerequisites:** MMathPhys students: B3 Quantum Atomic and Molecular Physics. MSc students: basic atomic physic. The lectures in weeks 1 - 2 of the course reviews the principles of atomic physics from first principles, presented in the B3 course. This is to ensure that students who enrol via the MSc route (external to the University) are brought up to date with those enrolling internally via the Oxford undergraduate physics course.

**Course Synopsis:**

Hot plasma is ubiquitous throughout the Universe and first appeared in the epoch of recombination that produced the cosmic background radiation about 378,000 years after the Big Bang. Since then quantum processes, particularly the emission and absorption of electromagnetic radiation from plasma, have provided essential information about the macroscopic structure of matter in the visible Universe. They are key to understanding stellar structure and evolution (along with helioseismology) by providing constraints on radiative transfer associated with nucleosynthesis of chemical elements in stellar interiors and in supernovae explosions. The effort to harness the immense power of nuclear fusion using magnetic or inertial confinement fusion schemes is being actively pursued world-wide. Indeed, these plasmas are among the most intense sources of X-rays in the laboratory and are used to study materials under extreme conditions of density and temperature. Emerging new tools, such as X-ray free electron lasers, are also being applied to these problems for the first time.

This course will introduce the student to the use of quantum mechanics in the computational modelling of hot plasmas. In the first part, an introduction to atomic processes is first provided to remind students of the basic principles of Slater’s configurational model and Racah’s tensor operator method. Then, the properties of electronic configurations and transition arrays are described, along with how they are used to replace the corresponding sets of individual levels and radiative lines. Following that, we will describe how these are applied to plasma dynamics and atomic processes, along with elegant new methods of super-configurations and effective temperatures. Finally, current applications are described, along with numerical and experimental examples.

**C7.5 GENERAL RELATIVITY I**

**Lecturer:** [**Dr Christopher Couzens**](https://www.maths.ox.ac.uk/people/christopher.couzens)

**Course Term:** Michaelmas

**Course Weight:** 1 unit/16 lectures

**Assessment Method:** written exam in TT

**General Prerequisites:** Special Relativity, Classical Mechanics and Electromagnetism

**Overview:**

The course is intended as an introduction to general relativity, covering both its observational implications and the new insights that it provides into the nature of spacetime and the structure of the universe. Familiarity with special relativity and electromagnetism as covered in the Part A and Part B courses will be assumed. The lectures will review Newtonian gravity, special relativity (from a geometric point of view), and then move on to cover physics in curved space time and the Einstein equations. These will then be used to give an account of planetary motion, the bending of light, the existence and properties of black holes and elementary cosmology.

**Course Synopsis:**

Introduction to the idea of “spacetime”. Review of Newtonian gravity. Review of Special Relativity, emphasising a geometric perspective. Difficulties in reconciling Special relativity with gravity, and the equivalence principle. Curved space time: elements of Lorentzian geometry, including vectors, co-vectors, tensors and their transformations; connections, curvature and geodesic deviation. The Einstein equations, and other physical laws in curved spacetime. Planetary motion and the bending of light. Introduction to black hole solutions; the Schwarzschild solution. Introduction to cosmology: homogeneity and isotropy, and the Friedman-Robertson-Walker solutions.

**C5.5 PERTURBATION METHODS**

**Lecturer:** [Prof Ruth Baker](https://www.maths.ox.ac.uk/people/ruth.baker)

**Course Term:** Michaelmas

**Course Weight:** 1 unit/16 lectures

**Assessment Method:** written exam in TT

**General Prerequisites:** Knowledge of core complex analysis and of core differential equations will be assumed, respectively at the level of the complex analysis in the Part A (Second Year) course Metric Spaces and Complex Analysis and the phase plane section in Part A Differential Equations I. The final section on approximation techniques in Part A Differential Equations II is highly recommended reading if it has not already been covered.

**Overview:**

Perturbation methods underlie numerous applications of physical applied mathematics: including boundary layers in viscous flow, celestial mechanics, optics, shock waves, reaction-diffusion equations, and nonlinear oscillations. The aims of the course are to give a clear and systematic account of modern perturbation theory and to show how it can be applied to differential equations.

**Course Synopsis:**

Introduction to regular and singular perturbation theory: approximate roots of algebraic and transcendental equations. Asymptotic expansions and their properties. Asymptotic approximation of integrals, including Laplace's method, the method of stationary phase and the method of steepest descent. Matched asymptotic expansions and boundary layer theory. Multiple-scale perturbation theory. WKB theory and semiclassics.

**C6.1 NUMERICAL LINEAR ALGEBRA**

**Lecturer:** [Prof Jared Tanner](https://www.maths.ox.ac.uk/people/jared.tanner)

**Course Term:** Michaelmas

**Course Weight:** 1 unit/16 lectures

**Assessment Method:** written exam in TT

**General Prerequisites:** Only elementary linear algebra is assumed in this course. The Part A Numerical Analysis course would be helpful, indeed some swift review and extensions of some of the material of that course is included here.

**Overview:**

Linear Algebra is a central and widely applicable part of mathematics. It is estimated that many (if not most) computers in the world are computing with matrix algorithms at any moment in time whether these be embedded in visualization software in a computer game or calculating prices for some financial option. This course builds on elementary linear algebra and in it we derive, describe and analyse a number of widely used constructive methods (algorithms) for various problems involving matrices.

**Course Synopsis:**

Common problems in linear algebra. Matrix structure, singular value decomposition. QR factorization, the QR algorithm for eigenvalues. Direct solution methods for linear systems, Gaussian elimination and its variants. Iterative solution methods for linear systems. Chebyshev polynomials and Chebyshev semi-iterative methods, conjugate gradients, convergence analysis, preconditioning.

**C3.1 ALGEBRAIC TOPOLOGY**

**Lecturer:** [Prof Andre Henriques](https://www.maths.ox.ac.uk/people/andre.henriques)

**Course Term:** Michaelmas

**Course Weight:** 1 unit/16 lectures

**Assessment Method:** written exam in TT

**General Prerequisites:** A3 Rings and Modules is essential, in particular a solid understanding of groups, rings, fields, modules, homomorphisms of modules, kernels and cokernels, and classification of finitely generated abelian groups.

A5 Topology is essential, in particular a solid understanding of topological spaces, connectedness, compactness, and classification of compact surfaces. B3.5 Topology and Groups is helpful but not necessary, in particular the notion of homotopic maps, homotopy equivalences, and fundamental groups will be recalled during the course. There will be little mention of homotopy theory in this course as the focus will be instead on homology and cohomology.

**Overview:**

Homology theory is a subject that pervades much of modern mathematics. Its basic ideas are used in nearly every branch, pure and applied. In this course, the homology groups of topological spaces are studied. These powerful invariants have many attractive applications. For example we will prove that the dimension of a vector space is a topological invariant and the fact that ‘a hairy ball cannot be combed’.

**Course Synopsis:**

Brief introduction to categories and functors. Applications of homology theory: Invariance of dimension, Brower fixed point theorem.

Chain complexes of free Abelian groups and their homology. Short exact sequences. of chain complexes, the induced long exact sequence in homology, and naturality. The snake lemma, the five lemma, splitting properties for short exact sequences.

Simplicial homology via Delta complexes.

Singular homology of topological spaces, and functoriality. Relative homology. Chain homotopies, homotopy equivalences. Homotopy invariance and excision (details of proofs not examinable). Retractions, deformation retractions, quotients.

Mayer-Vietoris Sequence. Wedge sums, cones, suspensions, connected sums.

Degree of a self-map of a sphere. Application: the hairy ball theorem.

Cell complexes and cellular homology. Equivalence of simplicial, cellular and singular homology.

Cochains and cohomology of spaces. Cup products.

Künneth Theorem (without proof). Euler characteristic. Ext and Tor groups via free resolutions. (Co)homology with different coefficients. The Universal Coefficient Theorem (proof not examinable).

Topological manifolds and orientability. The fundamental class of an orientable, closed manifold and the degree of a map between manifolds of the same dimension. Poincaré duality (proof not examinable). Manifolds with boundary and Poincaré-Lefschetz duality (proof not examinable). Brief discussion of locally finite homology, and cohomology with compact supports. Cap product.

Alexander duality. Applications: knot complements, Jordan curve theorem.

**C3.4 ALGEBRAIC GEOMETRY**

**Lecturer:** [Prof Damian Rossler](https://www.maths.ox.ac.uk/people/damian.rossler)

**Course Term:** Michaelmas

**Course Weight:** 1 unit/16 lectures

**Assessment Method:** written exam in TT

**General Prerequisites:** A3 Rings and Modules and B2.2 Commutative Algebra are essential. Noetherian rings, the Noether normalisation lemma, integrality, the Hilbert Nullstellensatz and dimension theory will play an important role in the course. B3.3 Algebraic Curves is useful but not essential. Projective spaces and homogeneous coordinates will be defined in C3.4, but a working knowledge of them would be useful. There is some overlap of topics, as B3.3 studies the algebraic geometry of one-dimensional varieties. Courses closely related to C3.4 include C2.2 Homological Algebra, C2.7 Category Theory, C3.7 Elliptic Curves, C2.6 Introduction to Schemes; and partly related to: C3.1 Algebraic Topology, C3.3 Differentiable Manifolds, C3.5 Lie Groups.

**Overview:**

Algebraic geometry is the study of algebraic varieties: an algebraic variety is roughly speaking, a locus defined by polynomial equations. One of the advantages of algebraic geometry is that it is purely algebraically defined and applied to any field, including fields of finite characteristic. It is geometry based on algebra rather than calculus, but over the real or complex numbers it provides a rich source of examples and inspiration to other areas of geometry.

**Course Synopsis:**

Affine algebraic varieties, the Zariski topology, morphisms of affine varieties. Irreducible varieties.

Projective space. Projective varieties, affine cones over projective varieties. The Zariski topology on projective varieties. The projective closure of affine variety. Morphisms of projective varieties. Projective equivalence.

Veronese morphism: definition, examples. Veronese morphisms are isomorphisms onto their image; statement, and proof in simple cases. Subvarieties of Veronese varieties. Segre maps and products of varieties.

Coordinate rings. The geometric form of Hilbert's Nullstellensatz. Correspondence between affine varieties (and morphisms between them) and finitely generated reduced K-algebras (and morphisms between them). Graded rings and homogeneous ideals. Homogeneous coordinate rings.

Discrete invariants of projective varieties: degree, dimension, Hilbert function. Statement of theorem defining Hilbert polynomial.

Quasi-projective varieties, and morphisms between them. The Zariski topology has a basis of affine open subsets. Rings of regular functions on open subsets and points of quasi-projective varieties. The ring of regular functions on an affine variety is the coordinate ring. Localisation and relationship with rings of regular functions.

Tangent space and smooth points. The singular locus is a closed subvariety. Algebraic re-formulation of the tangent space. Differentiable maps between tangent spaces.

Function fields of irreducible quasi-projective varieties. Rational maps between irreducible varieties, and composition of rational maps. Birational equivalence. Correspondence between dominant rational maps and homomorphisms of function fields. Blow-ups: of affine space at a point, of subvarieties of affine space, and of general quasi-projective varieties along general subvarieties. Statement of Hironaka's Desingularisation Theorem. Every irreducible variety is birational to a hypersurface. Re-formulation of dimension. Smooth points are a dense open subset.

**C3.3 DIFFERENTIABLE MANIFOLDS**

**Lecturer:** [Prof Dominic Joyce](https://people.maths.ox.ac.uk/joyce/)

**Course Term:** Michaelmas

**Course Weight:** 1 unit/16 lectures

**Assessment Method:** written exam in TT

**General Prerequisites:** A5: Topology and ASO: Introduction to Manifolds are strongly recommended. (Notions of Hausdorff, open covers, smooth functions on R^n will be used without further explanation.) Useful but not essential: B3.2 Geometry of Surfaces.

**Overview:**

A manifold is a space such that small pieces of it look like small pieces of Euclidean space. Thus a smooth surface, the topic of the Geometry of Surfaces course, is an example of a (2-dimensional) manifold.

Manifolds are the natural setting for parts of classical applied mathematics such as mechanics, as well as general relativity. They are also central to areas of pure mathematics such as topology and certain aspects of analysis.

In this course we introduce the tools needed to do analysis on manifolds. We prove a very general form of Stokes’ Theorem which includes as special cases the classical theorems of Gauss, Green and Stokes. We also introduce the theory of de Rham cohomology, which is central to many arguments in topology.

**Course Synopsis:**

Smooth manifolds and smooth maps. Tangent vectors, the tangent bundle, induced maps. Vector fields and flows, the Lie bracket and Lie derivative.

Exterior algebra, differential forms, exterior derivative, Cartan formula in terms of Lie derivative. Orientability. Partitions of unity, integration on oriented manifolds.

Stokes' theorem. De Rham cohomology. Applications of de Rham theory including degree.

Riemannian metrics. Isometries. Geodesics.

HILARY TERM

**ADVANCED FLUID DYNAMICS**

**Lecturers:** [Dr Paul Dellar](https://people.maths.ox.ac.uk/dellar/) and [Dr Andrew Mummery](https://www.physics.ox.ac.uk/our-people/mummery)

**Course Term:** Hilary

**Course Weight:** 1 unit/ 16 lectures

**Assessment Method:** written exam in week 0 TT **or** homework completion

**General Prerequisites:** Basic familiarity with fluid equations and stress tensors as provided, e.g., by Kinetic Theory

**Course Synopsis:**

**(**Part 1) Low Reynolds number hydrodynamics. The Stokes flow regime, general mathematical results, flow past a sphere. Stresses due to suspended rigid particles. Calculation of the Einstein viscosity for a dilute suspension. Stresses due to Hookean bead-spring dumb-bells. Derivation of the upper convected Maxwell and Oldroyd-B models for viscoelastic fluids. Properties of such fluids. Suspensions of orientable particles, Jeffery's equation, very brief introduction to active suspensions and liquid crystals.

(Part 2) Validity of the MHD approximation. Conservation equations. Magnetic force.  Evolution of the magnetic field. MHD waves. Static MHD equilibria. Relaxation.  MHD stability (normal modes, energy principle, application to a z-pinch). Non-ideal MHD.

**ADVANCED QUANTUM FIELD THEORY**

**Lecturer:** [Prof Prateek Agrawal](https://www.physics.ox.ac.uk/our-people/agrawal)

**Course Term:** Hilary

**Course Weight:** 1.5 units/24 lectures

**Assessment Method:** written exam in TT week 0

**General Prerequisites:** Quantum Field Theory (MT)

**Course Synopsis:**

Scalar QED: local gauge symmetry, path integrals, photon propagator, derivation of Feynman rules, radiative corrections. Fermions: spinors, Dirac equation, fermion propagator, derivation of Feynman rules, trace theorems and spinor technology. QED: symmetries and Feynman rules, scattering processes, radiative corrections and beta function. QCD and non-abelian gauge theory: introduction, gauge-fixing, Feynman rules, scattering processes, radiative corrections and beta function, BRST. Spontaneous symmetry breaking: global symmetries, abelian gauge symmetry, non-abelian gauge symmetry. The Standard Model: electroweak symmetry breaking and the Higgs mechanism, brief overview of particle content and interactions.

**ALGORITHMS AND COMPUTATIONS IN THEORETICAL PHYSICS: A SET OF LECTURES**

**Lecturer:** [Prof Werner Krauth](https://www.physics.ox.ac.uk/our-people/krauth)

**Course Term:** Hilary

**Course Weight:** 1 unit/16 lectures

**Assessment Method:** homework completion only

**Course Synopsis:**

This course introduces to algorithms and scientific computing from the viewpoint of statistical physics. It is also a practical, example-based, primer on subjects such as Markov chains, molecular dynamics, phase transitions, path integrals, superfluids and Bose-Einstein condensation, among others. The course stresses rigorous foundations and modern developments in mathematics (mixing, non-reversibility, perfect sampling,... ) and physics (entropic phase transition, KosterlitzThouless physics, cold atoms,... ), yet is entirely based on short Python programs. It will prepare advanced undergraduates in theoretical physics/math for the requirements of modern-day research, with the huge roles played by algorithmic thinking and by statistics, both with their power, their paradoxes and their intricacies.

**Contents:**

1. Sampling, Markov chains

1. Direct Sampling: From basic rejection sampling to Walker's method of aliases.

Sampling and (machine) learning.

1. Reversible & non-reversible Markov chains and equilibrium & out-of-equilibrium

statistical mechanics.

1. Mixing and relaxation times, the cutoff phenomenon.
2. Perfect sampling: Stopping rules, Coupling from the past.
3. Data analysis: From the strong law of large numbers to the ergodic theorem and to the bootstrap(s).

2. Classical hard disks and hard spheres

1. Molecular dynamics: From Alder and Wainwright (1957) to the heap-based

scheduling.

1. Hard-sphere Monte Carlo: From Metropolis et al. (1953) to the event-chain Monte

Carlo, piece-wise deterministic Markov chains.

1. Birth-and-death processes, perfect sampling.
2. Entropic phase transitions: melting.
3. One-dimensional simple exclusion models: Modern sampling algorithms in the light of rigorous mathematics.

3. Quantum Monte Carlo

1. Density matrices and path integrals
2. Superfluidity and condensate fractions: a path-integral point of view
3. Sampling of path integrals: from the Lévy construction to interacting path integrals
4. Ideal bosons from a path-integral perspective
5. Path integrals and random manifolds. Roughness exponents, fractional Brownian

motion.

4. Spin systems: samples and exact solutions

1. Ising Markov chains: From the Glauber dynamics to modern cluster algorithms.
2. Mixing and relaxation times: Modern mathematics and cutting-edge algorithms.
3. 2D XY model and the harmonic model: Reversible and non-reversible Markov-chain algorithms. Kosterlitz-Thouless physics.
4. Perfect sampling: semi-order
5. Modern sampling algorithms from the Onsager solution.

5. From Monte Carlo sampling to Machine learning

1. Inference and Bayesian statistics
2. Learning and classification

**Textbooks:**

1. W. Krauth, "Statistical Mechanics, Algorithms and Computations" (Oxford University Press, 2006; Second edition: 2023/24)

2. D. A. Levin and Y. Peres, "Markov Chains and Mixing Times (Second Edition)" (AMS, 2017)

3. L. Wasserman, "All of Statistics, A Concise Course in Statistical Inference" (Springer, 2005)

**COLLISIONLESS PLASMA PHYSICS**

**Lecturer:** [Dr Daniel Kennedy](https://www.physics.ox.ac.uk/our-people/kennedyd) and[Prof Alex Schekochihin](https://www.physics.ox.ac.uk/our-people/schekochihin)

**Course Term:** Hilary

**Course Weight:** 1 unit/18 lectures

**Assessment Method:** take-home exam **or** homework completion

**General Prerequisites:** Prequel: Kinetic Theory (MT), an undergraduate course on Electricity and Magnetism

**Areas:** CMT, Astro, foundational course

**Sequel:** Collisional Plasma Physics (TT) (note however that this course is self-contained and can be taken without continuing to Collisional Plasma Physics)

**Course Synopsis:**

Part I. Plasma waves:

Cold plasma waves in a magnetised plasma. WKB theory of cold plasma wave propagation in an inhomogeneous plasma, cut-offs and resonances. Hot plasma waves in a magnetised plasma. Cyclotron resonance.

Part II. Kinetics of strongly magnetised plasmas:

Kinetic description of a collisionless, magnetised plasma; kinetic MHD. Barnes damping, firehose and mirror instabilities. Particle motion. Drift kinetics. Drift waves and the ion-temperature-gradient instability. Electron drift kinetics (time permitting): kinetic Alfvén waves, electron-temperature-gradient instabilities**.**

**COSMOLOGY**

**Lecturer:** [Dr David Alonso](https://www.physics.ox.ac.uk/our-people/alonso)

**Course Term:** Hilary

**Course Weight:** 1 unit/16 lectures

**Assessment Method:** written exam in TT

**General Prerequisites:** General Relativity I (MT) or equivalent.

Einstein field equations and the Friedman equations, universe models, statistics of expanding background, relativistic cosmological perturbations, observations, from the Hubble flow to the CMB.

**GALACTIC AND PLANETARY DYNAMICS**

**Lecturer:** [Prof John Magorrian](https://www.physics.ox.ac.uk/our-people/magorrian)

**Course Term:** Hilary

**Course Weight:** 1 unit/16 lectures

**Assessment Method:** mini-project **or** homework completion

**Areas:** Astro

**General Prerequisites:** Kinetic Theory (MT)

**Course Synopsis:**

Review of Hamiltonian mechanics. Orbit integration. Classification of orbits and integrability. Construction of angle-action variables. Hamiltonian perturbation theory. Simple examples of its application to the evolution of planetary and stellar orbits. Methods for constructing equilibrium galaxy models. Applications. Fundamentals of N-body simulation. Dynamical evolution of isolated galaxies. Interactions with companions.

**GEOPHYSICAL FLUID DYNAMICS**

**Lecturer:** [Prof Tim Woollings](https://www.physics.ox.ac.uk/our-people/woollings)

**Course Term:** Hilary

**Course Weight:** 1 unit/16 lectures

**Assessment Method:** written exam in TT

**Course Synopsis:**

Rotating frames of reference. Geostrophic and hydrostatic balance. Pressure coordinates. Shallow water and reduced gravity models, f and β–planes, potential vorticity. Inertia-gravity waves, dispersion relation, phase and group velocity. Rossby number, equations for nearly geostrophic motion, Rossby waves, Kelvin waves. Linearised equations for a stratified, incompressible fluid, internal gravity waves, vertical modes. Quasigeostrophic approximation: potential vorticity equation, Rossby waves, vertical propagation and trapping. Eady model of baroclinic instability. Overview of large-scale structure and circulation of atmospheres and oceans, poleward heat transport. Angular momentum and Held-Hou model of Hadley circulations. Applications to Mars and slowly-rotating planets. Tide-locked exoplanets. Giant planets: Multiple jets, stable eddies and free modes.

**HIGH ENERGY DENSITY PLASMA PHYSICS**

**Lecturer:** [Prof Peter Norreys](https://www.physics.ox.ac.uk/our-people/norreys) and [Dr Ramy Aboushelbaya](https://www.physics.ox.ac.uk/our-people/aboushelbaya)

**Course Term:** Hilary

**Course Weight:** 1 unit/16 lectures

**Assessment Method:** homework completion only

**Course Synopsis:**

In this course, the topics will be introduced for first principles. The student will be taken through the fundamental physics of laser energy absorption in matter up to and including the new laser QED plasma regime at extreme intensities. The student will be introduced to hydrodynamic motion via first principles derivation of the Navier-Stokes equations as well as compression and rarefaction waves. Then a thorough grounding in hydrodynamic instabilities will be provided, including the Rayleigh-Taylor instability and the applications of linear theory. This will be followed by the extension to the convective instability; mode coupling; the Kelvin-Helmholtz; shock stability and the Richtmyer-Meshkov instability. The behaviour of shock waves in one dimension will then be discussed, including the derivation of the Rankine-Hugoniot equations; the effects of boundaries and interfaces; blast waves and shocks in solids. Following that, the physics of convergent shocks will be described. These include homogeneous expansion/contraction self-similar flows as well as shock dynamics. The hydrodynamic behaviour is governed by the equations of state including thermodynamic properties, so the student will be introduced to equations of state for gases, plasmas, solids and liquids. For thermal energy transport, the thermal energy transport equation is derived, as are the effects of the conductivity coefficients, inhibited thermal transport, electron-ion energy exchange, before electron degeneracy effects are introduced. The physics of radiation energy transport will be described, including radiation as a fluid and the Planck distribution function; radiation flux definition; solutions to the radiation energy transfer equations; material opacities; non-LTE radiation transport; radiation dominated hydrodynamics. Finally, dimensionless scaling laws for hydrodynamics will be outlined, ones that provide the student with a link between the fascinating detailed microphysics of laboratory plasma phenomena and exquisite astrophysical observations.

**NONEQUILIBRIUM STATISTICAL PHYSICS**

**Lecturer:** [Prof Ramin Golestanian](https://www.physics.ox.ac.uk/our-people/golestanian)

**Course Term:** Hilary

**Course Weight:** 1 unit/16 lectures

**Assessment Method:** mini-project **or** homework completion

**Areas:** CMT, Astro, foundational course

**Sequel:** Topics in Soft & Active Matter Physics (TT)

**Course Synopsis:**

Stochastic Langevin dynamics. Brownian motion. Nonequilibrium kinetics. Master equation. Fokker-Planck equation. Kramers rate theory and mean first-passage time. Brownian ratchets. Multiplicative noise. Path integral formulation and Martin-Siggia-Rose method. Fluctuation theorems.

**STRING THEORY I**

**Lecturer:** [Prof Xenia de la Ossa](https://www.maths.ox.ac.uk/people/xenia.delaossa)

**Course Term:** Hilary

**Course Weight:** 1 unit/16 lectures

**Assessment Method:** mini-project

**General Prerequisites:** Quantum Field Theory (MT)

**Course Synopsis:**

Historical background, Dolen-Horn-Schmid duality, the Veneziano and Virasoro-Shapiro amplitudes. Nambu-Goto and Polyakov world-sheet actions, equations of motion and constraints, open and closed strings and their corresponding boundary conditions. Old covariant quantization: the Virasoro algebra, physical state conditions, ghosts, critical spacetime dimension, and spacetime particle spectrum. Basic considerations of light-cone gauge quantization. Vertex operators and string scattering amplitudes. Strings in background fields, spacetime effective action. Circle compactification, elementary consideration of D-branes, T-duality.

**SUPERSYMMETRY AND SUPERGRAVITY**

**Lecturer:** [Dr Michele Levi](https://www.maths.ox.ac.uk/people/michele.levi)

**Course Term:** Hilary

**Course Weight:** 1 unit/16 lectures

**Assessment Method:** mini project

**Area:** PT

**Course Synopsis:**

The course includes the following chapters:

1. Context and Motivation.

2. Spinors Preliminary.

3. Supersymmetry Algebra.

4. Superspace and Superfields.

5. Chiral Superfields and Supersymmetric Actions.

6. Supersymmetric Gauge Theories.

7. Spontaneous Symmetry Breaking.

**QUANTUM MATTER**

**Lecturer:** [Professor Steve Simon](https://www-thphys.physics.ox.ac.uk/people/SteveSimon/)

**Course Term:** Hilary

**Course Weight:** 1 unit/16 lectures

**Assessment Method:** written exam in week 0 TT **or** homework completion

**Course Synopsis:**

Intro to Superfluids and Superconductors

Two Fluid Model and Vortices

Landau Criterion and Intro to Charged Superfluids / London Theory

Superconducting Vortices / Type I and Type II Superconductors

Microscopic Theory / Second Quantization / Gross Pitaevskii and Bogoliubov Theory

Feynman Theory of Superfluidity

Ginzburg Landau Theory / Anderson Higgs Mechanism / Coherence Length / Vortex Structure

Interacting Fermions / Second Quantization / First Order Perturbation Theory / Hartree and Fock Terms

Hartree Fock Theory

Coulomb Interaction / Screening and Response

Linear Response Theory / Lindhard, Thomas Fermi, RPA / Plasmons

Landau Theory of Fermi Liquids part 1

Landau Theory of Fermi Liquids part 2

BCS theory part 1: Phonon Attraction Mechanism / Cooper Problem

BCS wavefunction

Bogoliubov Excitation Spectrum

plus if time permits

Quantum Hall Effect / Laughlin Gauge Invariance Argument

Chern Numbers / Quantum Spin Hall / Topological Insulators in 2 and 3 D

Fractional Quantum Hall Effect

**C3.11 RIEMANNIAN GEOMETRY**

**Lecturer:** [Prof Andrew Dancer](https://www.maths.ox.ac.uk/people/andrew.dancer)

**Course Term:** Hilary

**Course Weight:** 1 unit/16 lectures

**Assessment Method:** written exam in TT

**General Prerequisites:** Differentiable Manifolds is required. An understanding of covering spaces will be strongly recommended.

**Overview:**

Riemannian Geometry is the study of curved spaces and provides an important tool with diverse applications from group theory to general relativity. The surprising power of Riemannian Geometry is that we can use local information to derive global results. This course will study the key notions in Riemannian Geometry: geodesics and curvature. Building on the theory of surfaces in R3 in the Geometry of Surfaces course, we will describe the notion of Riemannian submanifolds, and study Jacobi fields, which exhibit the interaction between geodesics and curvature. We will prove the Hopf--Rinow theorem, which shows that various notions of completeness are equivalent on Riemannian manifolds, and classify the spaces with constant curvature.The highlight of the course will be to see how curvature influences topology. We will see this by proving the Cartan--Hadamard theorem, Bonnet--Myers theorem and Synge's theorem.

**Course Synopsis:**

Riemannian manifolds: basic examples of Riemannian metrics, Levi-Civita connection.

Geodesics: definition, first variation formula, exponential map, minimizing properties of geodesics.

Curvature: Riemann curvature tensor, sectional curvature, Ricci curvature, scalar curvature.

Riemannian submanifolds: examples, second fundamental form, Gauss--Codazzi equations.

Jacobi fields: Jacobi equation, conjugate points.

Completeness: Hopf--Rinow and Cartan--Hadamard theorems

Constant curvature: classification of complete manifolds with constant curvature.

Second variation and applications: second variation formula, Bonnet--Myers and Synge's theorems.

**C5.4 NETWORKS**

**Lecturer:** [Prof Peter Grindrod](https://www.maths.ox.ac.uk/people/peter.grindrod)

**Course Term:** Hilary

**Course Weight:** 1 unit/16 lectures

**Assessment Method:** mini-project

**General prerequisites:** Basic notions of linear algebra, probability, dynamical systems, and some computational experience. The student may use the language of their choice for computational experiments. Relevant notions of graph theory will be reviewed and illustrated.

**Overview:**

Network Science provides generic tools to model and analyse systems in a broad range of disciplines, including biology, computer science and sociology. This course aims at providing an introduction to this interdisciplinary field of research, by integrating tools from graph theory, statistics and dynamical systems. Most of the topics to be considered are active modern research areas. This year the course has been altered to incorporate some new material on dynamically evolving networks and the analysis of scaling properties of growing networks.

**Course Synopsis:**

1. Preliminaries. Probability theory; Renewal processes; Random walks; Power-law distributions; Matrix algebra; Markov chains
2. Basic structural properties of networks. Network Definitions; Degree distribution; Measures from walks and paths; Clustering coefficient; Centrality Measures; Spectral properties
3. Random graph models. Random Graphs; Stochastic Block model; Configuration model; Small World model; Growing network with preferential attachment
4. Community detection: spectral method; Modularity
5. Dynamically evolving networks: Markov chains of random graphs, application to triadic closure, via a mean field analysis
6. Consensus dynamics
7. Random walks: Discrete-time random walks on networks; PageRank; Models of epidemics
8. Scaling properties of growing networks: sequentially combining networks to form large growing networks.

**C5.6 APPLIED COMPLEX VARIABLES**

**Lecturer:** [Prof Jon Chapman](https://www.maths.ox.ac.uk/people/jon.chapman)

**Course Term:** Hilary

**Course Weight:** 1 unit/16 lectures

**Assessment Method:** written exam in TT

**General Prerequisites:** The course requires second year core analysis (A2 complex analysis). It continues the study of complex variables in the directions suggested by contour integration and conformal mapping. A knowledge of the basic properties of Fourier Transforms is assumed. Part A Waves and Fluids and Part C Perturbation Methods are helpful but not essential.

**Overview:**

The course begins where core second-year complex analysis leaves off, and is devoted to extensions and applications of that material. The solution of Laplace's equation using conformal mapping techniques is extended to general polygonal domains and to free boundary problems. The properties of Cauchy integrals are analysed and applied to mixed boundary value problems and singular integral equations. The Fourier transform is generalised to complex values of the transform variable, and used to solve mixed boundary value problems and integral equations via the Wiener-Hopf method.

**Course Synopsis:**

Review of core complex analysis, analytic continuation, multifunctions, contour integration, conformal mapping and Fourier transforms.

Riemann mapping theorem (in statement only). Schwarz-Christoffel formula. Solution of Laplace's equation by conformal mapping onto a canonical domain; applications including inviscid hydrodynamics; Free streamline flows in the hodograph plane. Unsteady flow with free boundaries in porous media.

Application of Cauchy integrals and Plemelj formulae. Solution of mixed boundary value problems motivated by thin aerofoil theory and the theory of cracks in elastic solids. Reimann-Hilbert problems. Cauchy singular integral equations. Complex Fourier transform. Contour integral solutions of ODE's. Wiener-Hopf method.

**C3.5 LIE GROUPS**

**Lecturer:** [Prof Jason Lotay](https://people.maths.ox.ac.uk/lotay/)

**Course Term:** Hilary

**Course Weight:** 1 unit/16 lectures

**Assessment Method:** written exam in TT

**General Prerequisites:** ASO: Group Theory, A5: Topology and ASO: Multidimensional Analysis and Geometry are all useful but not essential. It would be desirable to have seen notions of derivative of maps from **R**n to **R**m, inverse and implicit function theorems, and submanifolds of **R**n. Acquaintance with the notion of an abstract manifold would be helpful but not really necessary.

**Overview:**

The theory of Lie Groups is one of the most beautiful developments of pure mathematics in the twentieth century, with many applications to geometry, theoretical physics and mechanics. The subject is an interplay between geometry, analysis and algebra. Lie groups are groups which are simultaneously manifolds, that is geometric objects where the notion of differentiability makes sense, and the group multiplication and inversion are differentiable maps. The majority of examples of Lie groups are the familiar groups of matrices. The course does not require knowledge of differential geometry: the basic tools needed will be covered within the course.

**Course Synopsis**:

Brief introduction to manifolds. Classical Lie groups. Left-invariant vector fields, Lie algebra of a Lie group. One-parameter subgroups, exponential map. Homomorphisms of Lie groups and Lie algebras. Ad and ad. Compact connected abelian Lie groups are tori. The Campbell-Baker-Hausdorff series (statement only).

Lie subgroups. Definition of embedded submanifolds. A subgroup is an embedded Lie subgroup if and only if it is closed. Continuous homomorphisms of Lie groups are smooth. Correspondence between Lie subalgebras and Lie subgroups (proved assuming the Frobenius theorem). Correspondence between Lie group homomorphisms and Lie algebra homomorphisms. Ado’s theorem (statement only), Lie’s third theorem.

Basics of representation theory: sums and tensor products of representations, irreducibility, Schur’s lemma. Compact Lie groups: left-invariant integration, complete reducibility. Representations of the circle and of tori. Characters, orthogonality relations. Peter-Weyl theorem (statement only).

Maximal tori. Roots. Conjugates of a maximal torus cover a compact connected Lie group. Weyl group. Reflections. Weyl group of *U(n).* Representations of a compact connected Lie group are the Weyl-invariant representations of a maximal torus (proof of inclusion only). Representation ring of maximal tori and *U(n).*

Killing form. Remarks about the classification of compact Lie groups.

**C3.2 GEOMETRIC GROUP THEORY**

**Lecturer:** [Prof Cornelia Drutu](https://www.maths.ox.ac.uk/people/cornelia.drutu)

**Course Term:** Hilary

**Course Weight:** 1 unit/16 lectures

**Assessment Method:** written exam in TT

**General Prerequisites:** Some familiarity with Cayley graphs, fundamental group and covering spaces (as for example in the course B3.5 Topology & Groups) would be a helpful though not essential prerequisite.

**Overview:**

The aim of this course is to introduce the fundamental methods and problems of geometric group theory and discuss their relationship to topology and geometry. The first part of the course begins with an introduction to presentations and the list of problems of M. Dehn. It continues with the theory of group actions on trees and the structural study of fundamental groups of graphs of groups.The second part of the course focuses on modern geometric techniques and it provides an introduction to the theory of Gromov hyperbolic groups.

**Course Synopsis:**

Free groups. Group presentations. Dehn's problems. Residually finite groups.

Group actions on trees. Amalgams, HNN-extensions, graphs of groups, subgroup theorems for groups acting on trees.

Quasi-isometries. Hyperbolic groups. Solution of the word and conjugacy problem for hyperbolic groups.

If time allows: Small Cancellation Groups, Stallings Theorem, Boundaries.

**C3.12 LOW-DIMENSIONAL TOPOLOGY AND KNOT THEORY**

**Lecturer:** [Prof Andras Juhasz](https://www.maths.ox.ac.uk/people/andras.juhasz)

**Course Term:** Hilary

**Course Weight:** 1 unit/ 16 lectures

**Assessment Method:** written exam in TT

**General Prerequisites:** B3.5 Topology and Groups (MT) and C3.1 Algebraic Topology (MT) are essential. We will assume working knowledge of the fundamental group, covering spaces, homotopy, homology, and cohomology. B3.2 Geometry of Surfaces (MT) and C3.3 Differentiable Manifolds (MT) are useful but not essential, though some prior knowledge of smooth manifolds and bundles should make the material more accessible.

**Overview:**

Low-dimensional topology is the study of 3- and 4-manifolds and knots. The classification of manifolds in higher dimensions can be reduced to algebraic topology. These methods fail in dimensions 3 and 4. Dimension 3 is geometric in nature, and techniques from group theory have also been very successful. In dimension 4, gauge-theoretic techniques dominate.This course provides an overview of the rich world of low-dimensional topology that draws on many areas of mathematics. We will explain why higher dimensions are in some sense easier to understand, and review some basic results in 3- and 4-manifold topology and knot theory.

**Course Synopsis:**

The definition of topological and smooth manifolds. Morse theory, handle decompositions, surgery. Every group can be the fundamental group of a manifold in dimension greater than three. The h-cobordism theorem, outline of proof and the Whitney trick. Application: The generalized Poincare conjecture.

Knots and links: Reidemeister moves, Seifert surface and genus, Alexander polynomial, fibred knots, Jones polynomial, prime decomposition, 4-ball genus.

3-manifolds: Heegaard decompositions, unique prime decomposition, loop theorem, lens spaces, Dehn surgery, branched double cover.

4-manifolds: Kirby calculus, the intersection form, Freedman’s and Donaldson’s theorems (without proof).

**C7.4 INTRO TO QUANTUM INFORMATION**

**Lecturer:** [Prof Artur Ekert](https://www.maths.ox.ac.uk/people/artur.ekert)

**Course Term:** Hilary

**Course Weight:** 1 unit/16 lectures

**Assessment Method:** written exam in TT

**General Prerequisites:** Quantum Theory. The course material should be of interest to physicists, mathematicians, computer scientists, and engineers. The following will be assumed as prerequisites for this course:

- elementary probability, complex numbers, vectors and matrices; - Dirac braket notation; - a basic knowledge of quantum mechanics especially in the simple context of finite dimensional state spaces (state vectors, composite systems, unitary matrices, Born rule for quantum measurements); - basic ideas of classical theoretical computer science (complexity theory) would be helpful but are not essential.

Prerequisite notes will be provided giving an account of the necessary material. It would be desirable for you to look through these notes slightly before the start of the course.

**Overview:**

The classical theory of computation usually does not refer to physics. Pioneers such as Turing, Church, Post and Goedel managed to capture the correct classical theory by intuition alone and, as a result, it is often falsely assumed that its foundations are self-evident and purely abstract. They are not! Computers are physical objects and computation is a physical process. Hence when we improve our knowledge about physical reality, we may also gain new means of improving our knowledge of computation. From this perspective it should not be very surprising that the discovery of quantum mechanics has changed our understanding of the nature of computation. In this series of lectures you will learn how inherently quantum phenomena, such as quantum interference and quantum entanglement, can make information processing more efficient and more secure, even in the presence of noise.

**Course Synopsis:**

Bits, gates, networks, Boolean functions, reversible and probabilistic computation

"Impossible" logic gates, amplitudes, quantum interference

One, two and many qubits

Entanglement and entangling gates

From interference to quantum algorithms

Algorithms, computational complexity and Quantum Fourier Transform

Phase estimation and quantum factoring

Non-local correlations and cryptography

Bell's inequalities

Density matrices and CP maps

Decoherence and quantum error correction

**C7.6 GENERAL RELATIVITY II**

**Lecturer:** [Dr Christopher Couzens](https://www.maths.ox.ac.uk/people/christopher.couzens)

**Course Term:** Hilary

**Course Weight:** 1 unit/16 lectures

**Assessment Method:** written exam in TT

**General Prerequisites:** C7.5 General Relativity I

**Overview:**

In this, the second course in General Relativity, we have two principal aims. We first aim to increase our mathematical understanding of the theory of relativity and our technical ability to solve problems in it. We apply the theory to a wider class of physical situations, including gravitational waves and black hole solutions. Orbits in the Schwarzschild solution are given a unified treatment which allows a simple account of the three classical tests of Einstein's theory. This leads to a greater understanding of the Schwarzschild solution and an introduction to its rotating counterpart, the Kerr solution. We analyse the extensions of the Schwarzschild solution show how the theory of black holes emerges and exposes the radical consequences of Einstein's theory for space-time structure.

**Course Synopsis:**

Mathematical background, the Lie derivative and isometries. The Einstein field equations with matter; the energy-momentum tensor for a perfect fluid; equations of motion from the conservation law. Linearised general relativity and the metric of an isolated body. Motion on a weak gravitational field and gravitational waves. The Schwarzschild solution and its extensions; Eddington-Finkelstein coordinates and the Kruskal extension. Penrose diagrams and the area theorem. Stationary, axisymmetric metrics and orthogonal transitivity; the Kerr solution and its properties; interpretation as rotating black hole.

**C7.7 RANDOM MATRIX THEORY**

**Lecturer:** [Prof Louis-Pierre Arguin](https://www.maths.ox.ac.uk/people/louis-pierre.arguin)

**Course Term:** Hilary

**Course Weight:** 1 unit/16 lectures

**Assessment Method:** written exam in TT

**General Prerequisites:** There are no formal prerequisites, but familiarity with basic concepts and results from linear algebra and probability will be assumed, at the level of A0 (Linear Algebra) and A8 (Probability).

**Overview:**

Random Matrix Theory provides generic tools to analyse random linear systems. It plays a central role in a broad range of disciplines and application areas, including complex networks, data science, finance, machine learning, number theory, population dynamics, and quantum physics. Within Mathematics, it connects with asymptotic analysis, combinatorics, integrable systems, numerical analysis, probability, and stochastic analysis. This course aims to provide an introduction to this highly active, interdisciplinary field of research, covering the foundational concepts, methods, questions, and results.

**Course Synopsis:**

Introduction to matrix ensembles, including Wigner and Wishart random matrices, and the Gaussian and Circular Ensembles. Overview of connections with Data Science, Complex Quantum Systems, Mathematical Finance, Network Models, Numerical Linear Algebra, and Population Dynamics (1 Lecture)

Statement and proof of Wigner’s Semicircle Law; statement of Girko’s Circular Law; applications to Population Dynamics (May’s model). (3 lectures)

Statement and proof of the Marchenko-Pastur Law using the Stieltjes and R-transforms; applications to Data Science and Mathematical Finance. (3 lectures)

Derivation of the Joint Eigenvalue Probability Density for the Gaussian and Circular Ensembles;

method of orthogonal polynomials; applications to eigenvalue statistics in the large-matric limit; behaviour in the bulk and at the edge of the spectrum; universality;

applications to Numerical Linear Algebra and Complex Quantum Systems (5 lectures)

Dyson Brownian Motion (2 lectures)

Connections to other problems in mathematics, including the longest increasing subsequence problem; distribution of zeros of the Riemann zeta-function; topological genus expansions. (2 lectures)

TRINITY TERM

**ADVANCED TOPICS IN PLASMA PHYSICS**

**Lecturer:** [Dr Daniel Kennedy](https://www.physics.ox.ac.uk/our-people/kennedyd)

**Course Term:** Trinity

**Course Weight:** 0.5 units/8 lectures

**Assessment Method:** homework completion only

**Area:** Fusion/astro plasma

**Course Synopsis:**

Basics of magnetic-confinement fusion. Magnetic geometry and flux surfaces in toroidal devices. Equilibrium vs fluctuations. Scale separation in time and space.

Asymptotic expansion the Vlasov-Landau equation. Gyrokinetic variables and gyroaverages. Derivation of the gyrokinetic equilibrium. Equilibrium Maxwell’s equations. Derivation of the gyrokinetic equation for plasma fluctuations. Fluctuating Maxwell’s equations. Free-energy conservation in gyrokinetics.

Plasma instabilities. Linear gyrokinetic theory and temperature-gradient-driven instabilities.

**AN INTRODUCTION TO TOPOLOGICAL PHASES OF MATTER**

**Lecturer:** [Prof Shivaji Sondhi](https://www.physics.ox.ac.uk/our-people/sondhi)

**Course Term:** Trinity

**Course Weight:** 1 unit/16 lectures

**Assessment Method:** mini-project

**Course Synopsis:** Single particle topology in solids, integer quantum Hall effect and topological insulators, fractionalization, topological order and gauge theories, topology in gapless matter (Chapters 1-7) of book.

**Textbook:** https://www.cambridge.org/core/books/topological-phases-of-matter/773DB63D42147A5703FF8BED94368D91

**ASTROPARTICLE PHYSICS**

**Lecturer:** [Prof Joseph Conlon](https://www.physics.ox.ac.uk/our-people/conlonj)

**Course Term:** Trinity

**Course Weight:** 1 unit/16 lectures

**Assessment Method:** homework completion only

**Pre-requisites:** Quantum Field Theory (MT), General Relativity I (MT).

**Syllabus:** The Universe observed, constructing world models, reconstructing our thermal history, decoupling of the cosmic microwave background, primordial nucleosynthesis. Dark matter: astrophysical phenomenology, relic particles, direct and indirect detection. Cosmic particle accelerators, cosmic ray propagation in the Galaxy. The energy frontier: ultrahigh energy cosmic rays and neutrinos. The early Universe: constraints on new physics, baryo/leptogenesis, inflation, the formation of large-scale structure, dark energy.

**COLLISIONAL PLASMA PHYSICS**

**Lecturer:** [Prof Sarah Newton](https://www.physics.ox.ac.uk/our-people/newtons)

**Course Term:** Trinity

**Course Weight:** 1 unit/16 lectures

**Assessment Method:** homework completion only

**Areas:** Astro

**Prequel:** Kinetic Theory (MT), Advanced Fluid Dynamics (HT), Collisionless Plasma Physics (HT)

**Course Synopsis:**

Collision operators: Fokker-Plank collision operator, conservation properties, entropy, electron-ion and ion-electron collisions, linearized collision operator.

Collisional transport (Braginskii equations: derivation of Spitzer resistivity and electron heat conduction, ion heat conduction and viscosity.

Resistive MHD: tearing modes, magnetic reconnection.

Introduction to tokamak theory: Pfirsch-Schlueter collision transport regime for electrons.

**CONFORMAL FIELD THEORY**

**Lecturer:** [Dr Romain Ruzziconi](https://www.maths.ox.ac.uk/people/romain.ruzziconi)

**Course Term:** Trinity

**Course Weight:** 1 unit/16 hours

**Assessment Method:** homework completion only

**General Prerequisites:** Quantum Field Theory (MT)

**Course Synopsis:**

* Motivation: RG flows and scale invariance.
* Conformal transformations.
* Consequence of conformal invariance.
* Radial quantization and the operator algebra.
* Conformal invariance in two dimensions.
* The Virasoro algebra.
* Minimal models.
* Conformal bootstrap in d > 2.

**DISORDER IN CONDENSED MATTER**

**Lecturer:** [Prof John Chalker](https://www.physics.ox.ac.uk/our-people/chalker)

**Course Term:** Trinity

**Course Weight:** 1 unit/16 lectures

**Assessment Method:** homework completion only

**General Prerequisites:** An understanding of many-body quantum mechanics including second quantization. An understanding of statistical mechanics to at least the level of a first undergraduate course. Knowledge of (or interest in) condensed matter physics to the level of a first undergraduate course.

**Course Synopsis:**

* Harmonic excitations in disordered systems
* Harris criterion, Imry-Ma phenomena, Griffiths effects
* Spin glasses
* Random matrix theory
* Anderson localisation
* Weak localisation, mesoscopics, hopping conduction in insulators, Coulomb glass
* Many-body localisation
* Infinite-disorder phases

**MACHINE LEARNING FUNDAMENTALS WITH APPLICATIONS TO PHYSICS AND MATHEMATICS**

**Lecturer:** [Dr Andrei Constantin](https://www.physics.ox.ac.uk/our-people/constantin)

**Course Term:** Trinity

**Course Weight:** 1 unit/16 lectures

**Assessment Method:** homework completion only

**Prerequisites:** Prior exposure to Mathematica and Python will be helpful, but not mandatory.

**Course Synopsis:**

Over the past five to ten years, machine learning and artificial intelligence in general, have evolved into indispensable research tools. This course seeks to offer a comprehensive introduction to the diverse array of machine learning techniques. These methods share the common goal of crafting algorithms that enable computers to make predictions and decisions autonomously, without relying on explicit, handcrafted rules. Using these techniques, one can extract valuable insights from computers that surpass the information initially provided.

The course will discuss fundamental principles, algorithms, and a number of applications in Mathematics and Physics, including state reconstruction in quantum physics, model building in particle physics and cosmology, applications to string theory compactifications, conformal bootstrap and knot theory.

1. Introduction to Machine Learning and Elements of Information Theory
2. Multi-Layer Perceptrons (MLPs) and Stochastic Gradient Descent (SGD)
3. Feed-Forward Computation Graphs and Backpropagation
4. Convolutional Neural Networks (CNNs)
5. Principal Component Analysis (PCA) and Invariance
6. Controlling Vanishing and Exploding Gradients: Initialisation, Batch Normalisation and Skip Connections
7. Stochastic Gradient Descent
8. Neural Networks for Supervised Learning
9. Clustering and Unsupervised Learning
10. Autoencoders
11. Generative Adversarial Networks
12. Reinforcement Learning
13. Solving Differential Equations with Neural Networks
14. Transformers
15. Genetic Programming
16. Symbolic Regression

**QUANTUM FIELD THEORY IN CURVED SPACE-TIME**

**Lecturer:** [Dr Pieter Bomans](https://www.maths.ox.ac.uk/people/pieter.bomans)

**Course Term:** Trinity

**Course Weight:** 1 unit/16 lectures

**Assessment Method:** homework completion only

**General Prerequisites:** Quantum Field Theory (MT), General Relativity I (MT) and General Relativity II (HT). Advanced Quantum Field Theory and a course on differential geometry will be helpful but not essential.

**Course Synopsis:**

This course builds on the courses in quantum field theory and general relativity. It will focus on classical and quantum aspects of fields in curved space-time. The course will consist of the following topics: Classical fields in curved space, Quantization in curved space, Quantum fields in (Anti) de Sitter space, Quantum fields in Rindler space and the Unruh effect, Hawking radiation, Black hole thermodynamics and the Hawking-Page phase transition, Interactions in curved space-time, Quantum field theory and cosmology.

**RENORMALIZATION GROUP**

**Lecturer:** [Prof Fernando Alday](https://www.maths.ox.ac.uk/people/luis.alday)

**Course Term:** Trinity

**Course Weight:** 1 unit/16 lectures

**Assessment Method:** homework completion only

**General Prerequisites:** Quantum Field Theory (MT), Statistical Mechanics (MT) or equivalent.

**Course Synopsis:**

This course introduces ideas of scale-invariance and the renormalisation group in statistical physics, using simple lattice models and field theories as examples. Topics include: Real space RG; Fixed points, scaling operators, operator product expansion etc.; Landau Ginsburg theory; Mean field theory; Large N approximation; the 4-epsilon expansion; the 2+epsilon expansion; the Kosterlitz-Thouless transition; The Sine-Gordon model; XY duality.

**STRING THEORY II**

**Lecturer:** [Prof Xenia de la Ossa](https://www.maths.ox.ac.uk/people/xenia.delaossa)

**Course Term:** Trinity

**Course Weight:** 1 unit/16 lectures

**Assessment Method:** homework completion only

**General Prerequisites:** String Theory I,Quantum Field Theory, and Advanced Quantum Field Theory for Particle Physics, Supersymmetry and Supergravity

**Course Synopsis:** Classical superstring action, RNS string, quantization and GSO-projection; 10d superstrings: Type IIA, IIB, I and Heterotic strings; Open strings and D-branes; Supergravities and spacetime effective actions, M-theory and 11d supergravity; Compactifications; Dualities between string theories.

**THE STANDARD MODEL AND BEYOND I**

**Lecturer:** [Prof Fabrizio Caola](https://www.physics.ox.ac.uk/our-people/caola)

**Course Term:** Trinity

**Course Weight:** 1 unit/ 16 lectures

**Assessment Method:** homework completion only

**Prerequisite:** Advanced Quantum Field Theory for Particle Physics (HT)

**Course Synopsis:**

Basics of strong interactions: the peculiarities of asymptotic freedom and the uniqueness of gauge theories

Low-energy effective actions: from QCD to the chiral Lagrangian, and Effective Field Theories

Building the Electroweak sector of the Standard Model

Exploring the structure of the Electroweak sector

QCD at colliders [if time permits]: OPE and factorisation, from hadrons to partons

You may find the following textbooks useful: H. Georgi, Weak Interactions and Modern Particle Theory; J.F. Donoghue, E. Golowich, Barry R. Holstein, Dynamics of the standard model.

**THE STANDARD MODEL AND BEYOND II**

**Lecturer:** [Prof John March-Russell](https://www.physics.ox.ac.uk/our-people/march-russell)

**Course Term:** Trinity

**Course Weight:** 1 unit/16 lectures

**Assessment Method:** homework completion only

**Area:** PT

**Prerequisite:** Advanced Quantum Field Theory for Particle Physics (HT)

**TOPICS IN SOFT AND ACTIVE MATTER PHYSICS**

**Lecturer:** [Prof Ard Louis](https://www.physics.ox.ac.uk/our-people/louis)

**Course Term:** Trinity

**Course Weight**: 0.5 units/8 lectures

**Assessment Method:** homework completion only

**Area:** CMT

**Prequels:** Advanced Fluid Dynamics (HT)

**Course Synopsis:**

This is a reading course. Under the guidance of the course organiser, students will give presentations based on key papers in soft condensed matter theory. Some examples of the topics for these presentations are: Active nematics and active gels. Wetting, spreading and contact line dynamics. Hydrodynamics of microswimmers: Stokes equation, scallop theorem, multipole expansion, active suspensions. Fluctuations and response.