Honour School of Mathematical and Theoretical Physics Part C
Master of Science in Mathematical and Theoretical Physics

GEOPHYSICAL FLUID DYNAMICS

Trinity Term 2025

Monday 9 June 2025, 14:30 - 16:30

You should submit answers to both questions.

You must start a new booklet for each question which you attempt. Indicate on the front sheet the numbers of the questions attempted. A booklet with the front sheet completed must be handed in even if no question has been attempted. Calculators are permitted.

The numbers in the margin indicate the weight that the Examiners anticipate assigning to each part of the question.

Do not turn this page until you are told that you may do so

1. (a) [6 marks] Make a basic sketch of Earth's radiation balance as a function of latitude, and use this to argue why the atmosphere and ocean have to circulate. In the tropics, this circulation is dominated by the Hadley cell. Assuming an air parcel starts at rest over the equator, calculate its zonal velocity at the poleward extent of the Hadley cell, which can be assumed to be at 30°N.

(b) [6 marks] Assuming geostrophic and hydrostatic balance, derive the thermal wind equation for the zonal wind u in height coordinates

$$f\frac{\partial u}{\partial z} \approx -\frac{g}{T}\frac{\partial T}{\partial y},$$

and comment on how good an approximation this is. (The quantities above have their usual meaning.) Hence estimate the meridional temperature difference across the subtropical jet stream which lies at the Hadley cell edge.

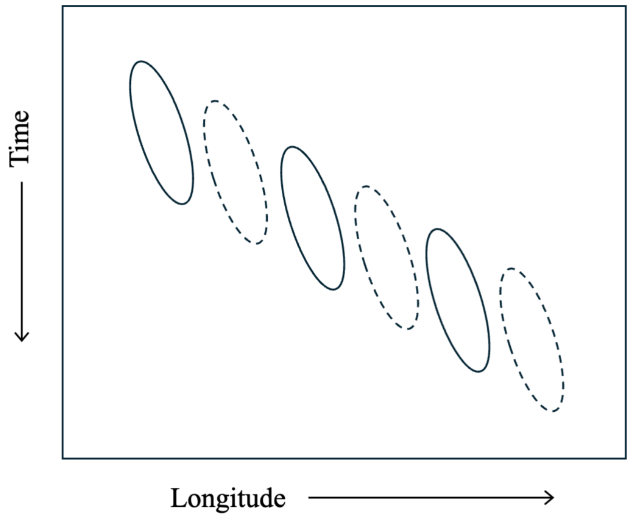
(c) [7 marks] The simplest system capable of supporting Rossby waves is incompressible flow in a single layer of fixed depth on a β-plane, represented by

$$\left(\frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y}\right)\xi + \beta v = 0,$$

where ξ is the relative vorticity. Linearise this equation about a background constant westerly wind U. Then show that there are wave solutions with dispersion relation

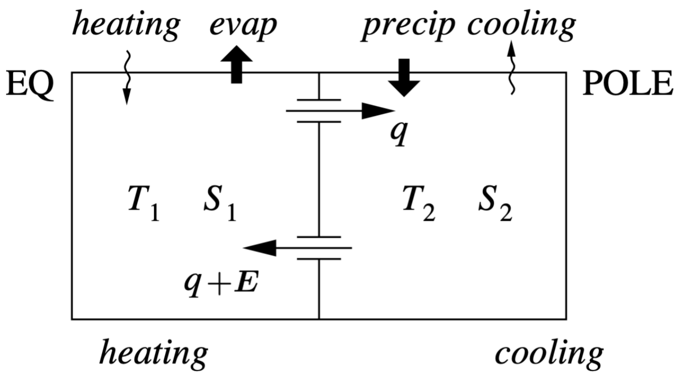
$$\omega = Uk - \frac{\beta k}{k^2 + l^2},$$

where $\omega \geqslant 0$ is the frequency and k and l are the zonal and meridional wavenumbers respectively. The figure below shows an idealised zonally propagating linear wave packet represented by the meridional wind v (negative contours dashed) as a function of longitude and time. Based on your solution, determine whether the wavenumber k is greater or less than $(\beta/U)^{\frac{1}{2}}$.



(d) [6 marks] In reality, the background wind is not constant, and hence the variation of zonal wind with latitude gives a relative vorticity gradient in addition to the planetary vorticity gradient. Estimate the magnitude of the two vorticity gradients for the subtropical jet that you calculated above. Determine qualitatively how the phase and group velocities, and hence the wave packet in the figure would be altered as a result, assuming the same zonal wavenumber.

2. (a) [7 marks] A simple model of the Atlantic Meridional Overturning Circulation consists of two well-mixed volumes of water connected by pipes representing upper and lower level volume flux q. The warm box has temperature T_1 and salinity S_1, and the cold box has temperature T_2 and salinity S_2. Fresh water is removed from the warm box by evaporation (volume flux E) and returned to the cold box by precipitation at the same rate. A volume flux E back into the warm box at depth is then included to balance the fluxes. The densities are given by the equation of state $\rho = \rho_0(1-\alpha T+\beta S)$, where α and β are positive constants.



If the volume flux is given by $q = k(\rho_2 - \rho_1)/\rho_0$, show that q satisfies an equation of the form

$$q|q| - A|q| + BE = 0.$$

Obtain expressions for the coefficients A and B in terms of the temperature difference $\Delta T = T_1 - T_2$, a reference salinity S_0 and the parameters given.

(b) [10 marks] Given $k = 7 \times 10^6\,\mathrm{Sv}$, $E = 0.3\,\mathrm{Sv}$, $\Delta T = 15\,\mathrm{K}$, $S_0 = 35\,\mathrm{g\,kg^{-1}}$, $\alpha = 2 \times 10^{-7}\,\mathrm{K^{-1}}$ and $\beta = 8 \times 10^{-7}\,\mathrm{kg\,g^{-1}}$ in the present day, deduce values for A and B. Hence, produce a labelled sketch of the variation of possible steady state solutions for q as a function of E, indicating without proof which solutions are stable and unstable. Estimate the present-day strength of overturning q and the critical value of E when the solution becomes single-valued. Comment on the nature of your solutions and discuss what would happen if E were to slowly increase from its present value, and what implications this would have.

(c) [4 marks] Many different dynamical processes affect the overturning circulation. Specifically, what is the role of i) northern high-latitude cooling, ii) entrainment in marginal sea overflows, iii) Southern Ocean winds and iv) rough deep sea topography in the mid-Atlantic?

(d) [4 marks] Given these complexities, it is not surprising that there is considerable uncertainty over the parameter k in this model. Suppose that a new discovery were to suggest a reduction to λk, where $0 < \lambda < 1$. How would your solution sketch change qualitatively, and what would be the implications?

[$1\,\mathrm{Sv} = 10^6\,\mathrm{m^3 s^{-1}}$]